LONG-RUN RISK, COINTEGRATION, AND THE DISTRESS PUZZLE

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Abstract. The *distress puzzle* refers to the empirical regularity that stocks with high measures of default likelihood earn anomalously low returns, despite having relatively high CAPM betas. This paper shows that it is possible to qualitatively explain this anomaly using a consumption-based asset pricing model, related to the literature on long-run risks. Apart from the usual assumptions of time varying, persistent conditional mean and volatility of consumption and dividend growth rates, I assume cointegration between the levels of consumption and aggregate dividend. To model distress, I employ a simple intensity-based model of default. Distressed firms have short expected lifetimes, so they do not covary with the long-run risk factors, and thus earn low expected returns. Healthy firms are long-lived in expectations, and because of the cointegrating relation, their prices do not respond very strongly to current innovations in aggregate dividend, which lowers their market betas. The model is calibrated to match the standard set of stock market moments and reproduces them well.

1. Introduction

The *distress puzzle* is an asset pricing anomaly that has been very difficult to explain by means of risk-based models with rational agents. Empirically, stocks with high measures of distress earn low returns, despite they tend to load high on the market factor (and also on the HML and SMB factors of Fama and French [33]).[1]

The problem is sometimes re-stated in the following way: is default an aggregate factor that is priced by the markets? If yes, stocks with high default probability should earn on average higher returns because of the higher loading. Again, if this was the case, the factor constructed on the basis of the difference between the returns of the most and the least distressed firms should bear positive price of risk. It has been documented that the opposite holds, and the market price of distress risk shows up with a puzzling negative sign. In a seminal paper, Dichev [28] constructs a distress factor based on models of bankruptcy prediction of Altman [1] and Ohlson [52], and shows that high bankruptcy risk is not rewarded by high returns. Moreover, in his post-1980 subsample, the relation is significantly negative.[2] In a more recent study, Campbell, Hilscher and Szilagyi [24] perform a similar analysis, sorting firms into groups according to fitted default probabilities from a probit regression model. The results are even more dramatic – the most distressed group earns an average annual return of minus 16%.[3] Similar qualitative results are reported by Griffin and Lemmon [38], Garlappi, Shu and Yan [35], and Breig and Elsas [18] (in a study of German firms).

In this paper, I show that the puzzle can be at least partially understood within a consumption-based model that features a representative agent with Epstein-Zin utility function (see Epstein and Zin [31]), and a small but persistent component in the growth rate of consumption, which is essentially the long-run risks framework of Bansal and Yaron [14]. Innovations to this component bear high prices of risk, and the model generates considerable equity premium through the covariation of long-term equity with the risk

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[1] Interestingly, anomalously low returns were also reported for distressed *bonds*, see Altman [2].

[2] This result holds despite *positive* correlation of default likelihood with size and book to market. Fama and French [32] conjectured that the book-to-market effect might be due to the risk of financial distress. The result of Dichev provided evidence that this is not the case.

[3] These results are robust to the survival bias, which works in the opposite direction.

factor.[4] Since distressed firms are necessarily those that have short expected maturity of cash-flows, they cannot earn high returns in the model, which is the first step to the resolution of the distress puzzle in my paper.

To make the explanation complete, one also needs to account for high market betas of the distressed stocks. To achieve this goal, I assume that consumption and dividends are cointegrated, which assures that covariation of long-maturity asset prices with the innovations to current dividend growth is relatively low: when a large positive dividend comes, it is usually not matched by high consumption growth (because of its much smaller volatility). Thus, the increase in the dividend will be reverted in the long run due to the cointegration, and the prices of long maturity assets will respond to such innovation relatively weakly, which means that the part of their market beta that comes from exposure to current cash flow innovations is small. The story goes the other way when one considers the part of market beta that is due to the innovations in the long-run risk factors – long-term equity covaries with them more strongly than short-term equity. Taken together, the slope of market beta as a function of maturity is ambiguous, but under my calibration the first effect dominates, and distressed stocks load higher on the market. I find that even small amount of cointegration can produce this effect, and that it can be achieved without damaging the ability of the model to match the standard set of data moments that is usually the target of calibration in the long-run risks literature.

To model distress, I employ a simple intensity-based model of default, in which firms are indexed by constant probability of default per period of time. To make the idea as clear as possible, I assume that the events of distress are uncorrelated with the pricing factors.[5] This also considerably simplifies the analysis, because I do not need to model financial leverage (the cash flows can be interpreted as flowing directly to equity). In the event of default, a firm is assumed to pay its last dividend in the following period, and its ex-dividend price falls to zero.

The model is calibrated to match a set of the most important data moments, which are the means, volatilities and first-order autocorrelations of consumption and dividend growth rates, realized equity premium, risk-free rate and the price-dividend ratio. My calibration is close to Bansal and Yaron [14] in terms of the persistence of the long-run risks factors. With calibrated parameters, I compute expected excess returns of ten groups of firms with various probabilities of default. I chose the probabilities to be exactly the same as in the study of Campbell, Hilscher and Szilagyi [24], to have direct comparison to empirical results. The model qualitatively reproduces the facts very well, but has problems in matching them quantitatively. The magnitude of the puzzle in the data is too high.

This study fits into the growing literature on the possible explanations of the distress anomaly. Griffin and Lemmon [38] relate its size to indicators of informational asymmetry. Campbell et al. [24] consider two explanations: markets may be irrational or inefficient. Livdan, Sapriza and Zhang [49] provide a model in which financial constraints leads firms to decrease investment, which lowers risk and expected returns. Garlappi et al. [35] consider possibility that equity holders possess an American option to a fixed portion of assets, if absolute priority rule is violated upon bankruptcy. George and Hwang [36] use a model with market frictions to show that when financial distress is costly and firms make optimal capital structure decisions, *low* leverage firms will be endogenously exposed to high systematic risk. The three latter studies share one shortcoming, the low riskiness of distressed equity is well explained by traditional risk measures. Von Kalckreuth [44] argues that the incentives to withdraw resources from a firm as private benefits are larger if default probability is high, and the benefits are included in the price before they are extracted. Finally, Avramov, Cederburg and Hore [6], similarly to this study, use a (continuous time) version of the long-run risks model in an attempt to explain three anomalies, including

[4]The model also features another channel for generating high risk premia – shocks to the *volatility* of consumption growth.

[5]Some support for this assumption is provided by Opler and Titman [53], and Asquith, Gertner and Scharfstein [5], who show that bankruptcies are mostly due to idiosyncratic reasons. From another perspective, even if economic conditions play some role, it can be argued that defaults are related to changing economic conditions only at business cycle frequency. Bansal, Kiku and Yaron [11] show that business-cycle related risk is only weakly priced by long-run risks models.

the distress puzzle.[6] However, distressed stocks in their model have lower betas, which makes the explanation incomplete.

Another strand of literature to which my work is related investigates the implications of the long-run risk factors on asset prices. Examples are: Bansal and Yaron [14], [15], Bansal, Dittmar and Lundblad [8], Kiku [45], Bansal, Kiku and Yaron [9], Hansen, Heaton and Li [41], Bansal and Shaliastovich [13], Beeler and Campbell [16] and Bhamra, Kuehn and Strebulaev [17]. The paper that is the closest to mine in terms of modeling approach is Drechsler [30], who combines a cointegrated long-run risks model of Bansal and Yaron [15] with the approach of Lettau and Wachter [47], to obtain expected returns and conditional alphas and betas of *equity strips*, which are claims to aggregate dividends paid at various maturities. He concludes that under his calibrations the model cannot account for the *value premium* puzzle when firms differ only in payout horizons – in other words, the CAPM seems to work too well. He does not consider the possibility that the framework can produce substantial failure of the CAPM for very short-horizon equity strips, which idea I pursue here.

The paper is organized as follows: sections 2 and 3 present the model and the solution method. Section 4 provides the details on calibration, section 5 gives the results that obtain when the model is applied to distressed firms, and section 6 concludes. The appendix provides some proofs and technical details.

2. The Long-Run Risks Model

The exposition of the long-run risks model in this section is similar to the one in Bansal and Yaron [14] or Beeler and Campbell [16]. I start with the description of the representative consumer's utility function and equilibrium stochastic discount factor. Then, I characterize the price-consumption ratio for aggregate consumption claim[7] and the price-dividend ratio for the aggregate market. The two ratios are central to the derivation of asset pricing relations. Next, I show how to decompose the market price-dividend ratio to account for differences in payout horizons of single *equity strips*, that is, I solve for the prices of aggregate dividends paid at fixed time horizons. Finally, I solve for the excess returns and market betas.

2.1. **The stochastic discount factor.** A representative agent maximizes her lifetime utility, given by the recursive formulation,

$$V_t = \left[(1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left(E_t \left[V_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}},$$

where $0 < \delta < 1$ is the subjective time-discount factor, C_t is consumption at time t, γ is the coefficient of relative risk aversion (RRA), and θ is defined as $(1-\gamma)/(1-1/\psi)$, where ψ is the elasticity of intertemporal substitution (EIS). V_t depends on future values of itself through recursive specification. The term $\left(E_t\left[V_{t+1}^{1-\gamma}\right]\right)^{1/\theta}$ is the certainty equivalent at time t of random utility index at $t+1$, which depends on agent's aversion towards atemporal risk. The certainty equivalent is combined with time-t consumption through the time aggregator – a CES function with elasticity of substitution given by the EIS.[8]

It is assumed that the wealth of the agent evolves according to the budget constraint

$$W_{t+1} = (W_t - C_t) R_{c,t+1},$$

where W_t is her wealth and $R_{c,t+1}$ is the return on the claim to aggregate consumption. Since consumption and aggregate dividend are modeled as separate processes (as will be seen below), one must make an implicit assumption that the wealth has a component that

[6]The other two puzzles refer to the negative relations between returns and idiosyncratic volatility, found by Ang, Hodrick, Xing and Zhang [3], [4], and between returns and dispersion in earnings forecasts, reported by Diether, Malloy and Scherbina [29]. The choice of continuous time is motivated by the need to model the share of the aggregate dividend in a way similar to Menzly, Santos and Veronesi [51].

[7]Price-consumption ratio can also be referred to as wealth-consumption ratio, since the price in the numerator is risk-adjusted discounted stream of future consumption.

[8]This form of utility, proposed by Epstein and Zin [31], is designed to make a distinction between coefficients of risk aversion and elasticity of intertemporal substitution, which is helpful in resolving the equity premium puzzle. For a very nice exposition on this and other forms of non-standard utility functions and their applications in economics, see Backus, Routledge and Zin [7].

is the difference between the value of aggregate consumption and aggregate dividend. It is also assumed that the risk-free asset is in zero supply so that all wealth that is not consumed must be invested in risky asset that earns return $R_{c,t+1}$.

Consumption process is given exogenously (see below). The equilibrium marginal rate of substitution between date t and $t+1$ can be shown to be[9]

$$M_{t+1} = \delta^{\theta}(C_{t+1}/C_t)^{-\theta/\psi} R_{c,t+1}^{\theta-1}.$$

Since the model turns out to have convenient log-normal structure, it is more practical to work with the logarithm of M_{t+1}. I define $m_{t+1} \equiv log(M_{t+1})$,

$$m_{t+1} = \theta log(\delta) - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \tag{1}$$

where $r_{c,t+1} = log(R_{c,t+1})$ and Δc_{t+1} is a logarithmic growth rate of the aggregate consumption. I will refer to m_{t+1} as log stochastic discount factor (log SDF). Every asset with generic logarithmic (continuously compounded) return r^x_{t+1} must satisfy the standard no-arbitrage condition,

$$E_t[e^{m_{t+1}+r^x_{t+1}}] = 1. \tag{2}$$

Although the recursive formulation of Epstein-Zin utility function does not show up in equation (1), it is there implicitly, because of the term $r_{c,t+1}$. The representative consumer defines "good times" not only in terms of the strength of consumption growth one period ahead, but also through the return on her wealth. This opens up a channel for non-zero price of risk associated with any shock that is orthogonal to *current* consumption growth, but affects *future* consumption growth or its *riskiness*. All such shocks are reflected in the innovations to $r_{c,t+1}$. Note also, that when $\gamma = 1/\psi$, then $\theta = 1$ and $r_{c,t+1}$ is not present in the log SDF formula anymore. In this case, the Epstein-Zin utility function reduces to the standard power utility.

2.2. **Aggregate consumption and dividends.** Define Δc_{t+1} and Δd_{t+1} as logarithmic growth rates of, respectively, aggregate consumption and dividend. They are assumed to have the following dynamics

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + \psi_c x_t + \phi_c y_t + \sigma_t \eta_{t+1} \\
\Delta d_{t+1} &= \mu_d + \psi_d x_t + \phi_d y_t + \varphi \sigma_t u_{t+1} \\
x_{t+1} &= \rho x_t + \varphi_x \sigma_t \varepsilon_{t+1} \\
y_t &\equiv d_t - c_t \\
y_{t+1} &= (\mu_d - \mu_c) + (\psi_d - \psi_c) x_t + (1 + \phi_d - \phi_c) y_t + \sigma_t(\varphi u_{t+1} - \eta_{t+1}) \\
\sigma^2_{t+1} &= (1-\nu)\sigma^2 + \nu \sigma^2_t + \sigma_w w_{t+1} \\
corr(\eta, u) &= \alpha; \quad \phi_d < 0 < \phi_c
\end{aligned} \tag{3}$$

If consumption growth were i.i.d., then the returns on the wealth portfolio $r_{c,t+1}$ in (1) would be perfectly correlated with innovations to consumption growth. The log SDF would trivialize and attach non-zero price of risk only to current consumption shocks. In the above formulations, this is not true because there are three state variables whose innovations potentially affect the return on the wealth portfolio without directly affecting current consumption growth.

The variable x_t has the interpretation of a small, but persistent component in the growth rates of consumption and dividends. Similarly, σ^2_t is proportional to conditional variances of innovations to all variables other than itself.[10] Finally, y_t is defined as the discrepancy from a unit cointegrating relation between consumption and dividends.[11] The assumption $\phi_d < 0 < \phi_c$ is necessary to assure stationarity of this discrepancy series, given

[9] See Cochrane [26], the appendix.

[10] It is actually possible to replace the constant σ_w with a square root of σ^2_t (scaled by a constant). Time-varying variance would then provide volatility for its own innovation, which would be likely to change the properties of volatility risk in a potentially interesting way. The solution of the model would require solving a quadratic equation at the stage of undetermined coefficient method for the price-consumption ratio.

[11] Unit cointegration means that in the long run all dividends have to be consumed and only dividends can be consumed. This is not restrictive here, because adding a constant to the cointegrating relation does not change dynamic properties of y_t.

that cointegration does matter in the model. It also has an economic interpretation – if consumption is far below dividends, its growth is expected to accelerate. If dividends are above consumption – their growth will be lower in the future.[12]

In the whole system, there are four shocks $(\eta, u, \varepsilon, w)$, among which only the first two are correlated, with coefficient α. This is mainly for the sake of parsimony. The constants have rather intuitive interpretations. μ_d and μ_c are the fixed parts of expected consumption and dividend growth rates. ψ_c and ϕ_c are the sensitivities of consumption growth rate with respect to the state variables x_t and y_t. Analogous interpretation holds for ψ_d and ϕ_d. ρ and ν are the autocorrelation coefficients of x_t and σ_t^2. φ and φ_x determine the conditional volatilities of the dividend growth Δd_{t+1} and innovation to x_t, relative to the volatility of consumption growth. σ_w governs the conditional volatility of shocks to the variance σ_2. Parameter values are chosen in the next section to match the most important moments of consumption and dividend growth rates, as well as those of the risk-free rate, market excess return and the price-dividend ratio.

The system of equations (3) is very similar to that studied by Bansal and Yaron [15] and Drechsler [30]. Bansal and Yaron specify the process of the discrepancy from the cointegrating relation directly, rather than deriving it from consumption and dividend processes. They also do not consider a potential feedback from the cointegrating relation on consumption growth, setting $\phi_c = 0$. Drechsler [30] assumes a richer correlation structure between the shocks, which is not necessary for the main point of my paper.

3. Model Solution

3.1. **Log-linear approximations.** The model can be solved in an approximate closed form, using the Campbell and Shiller [23] approximation for the realized returns. Consider the stream of aggregate dividends $D_s, s \in \{\text{t,t+1,...}\}$, whose expected discounted sum from $t+1$ to infinity is P_t (ex dividend price). Let $z_{d,t}$ denote the log price-dividend ratio $log(P_t/D_t)$, Δd_{t+1} the logarithmic growth rate of dividends and $r_{d,t+1}$ the log gross return. Then,

$$r_{d,t+1} = \kappa_{d,0} + \Delta d_{t+1} + \kappa_{d,1} z_{d,t+1} - z_{d,t}, \tag{4}$$

where $\kappa_{d,0}$ and $\kappa_{d,1}$ are constants of linearization, given by[13]

$$\begin{aligned} \kappa_{d,1} &= \frac{exp(\bar{z}_d)}{1 + exp(\bar{z}_d)}, \\ \kappa_{d,2} &= log(1 + exp(\bar{z}_d)) - \kappa_{d,1}\bar{z}_d, \end{aligned} \tag{5}$$

and $\bar{z}$ is the mean log price-dividend ratio. Campbell [19] and Campbell and Koo [22] find that this approximation is highly accurate for models with constant volatility, provided that an iteration is used to find a fixed point for $\bar{z}$ (as will be the case here – see below). Kiku [45] and Bansal and Shaliastovich [13] compare approximate and numerical solutions for their long-run risks models, concluding that the differences between the two approaches are negligible. Since my calibration does not depart radically from theirs, I assume that this is true for my model as well, and rely on approximate solutions.

An approximation similar to 4 can be applied to the aggregate consumption claim that appears in the formula (1) for the log SDF. With analogous notation, we have

$$r_{c,t+1} = \kappa_{c,0} + \Delta c_{t+1} + \kappa_{c,1} z_{c,t+1} - z_{c,t}, \tag{6}$$

$$\begin{aligned} \kappa_{c,1} &= \frac{exp(\bar{z}_c)}{1 + exp(\bar{z}_c)}, \\ \kappa_{c,2} &= log(1 + exp(\bar{z}_c)) - \kappa_{c,1}\bar{z}_c, \end{aligned} \tag{7}$$

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[12] This assumption also eliminates the possibility that $\phi_d = \phi_c$, which would imply unit root in y.

[13] To derive 4, write

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{\left(\frac{P_{t+1}}{D_{t+1}} + 1\right) D_{t+1}}{P_t} = \left(\frac{P_{t+1}}{D_{t+1}} + 1\right) \frac{D_t}{P_t} \frac{D_{t+1}}{D_t} = (exp(z_{d,t+1}) + 1) \frac{D_t}{P_t} \frac{D_{t+1}}{D_t},$$

take logarithms and linearize $log(exp(z_{d,t+1}) + 1)$ around the mean price-dividend ratio $\bar{z}_d$, using the first-order Taylor expansion.

where $z_{c,t}$ is the log of the price-consumption ratio, and $\bar{z}_c$ its long-run mean.[14]

The constant $\kappa_{c,0}$ can be shown to be a number slightly above zero and $\kappa_{c,1}$ is smaller than one. $\kappa_{d,0}$ and $\kappa_{d,1}$ have similar properties. All of them are obtained numerically (see the details on the price-consumption and price-dividend ratios below, and the appendix).

Armed with the formulas for realized returns to aggregate claims (4) and (6), it is easy to obtain analytical expressions for the expected returns of all assets within in the model. In principle, this can be accomplished using the no-arbitrage condition (2), together with the log SDF (1). The only missing things are the expressions for the log valuation ratios $z_{c,t}$ and $z_{d,t}$ (as functions of x_t, y_t, σ_t^2), and the constants in (5) and (7).

3.2. **Price-consumption ratio.** Since the price-consumption ratio is necessary for the log SDF, one needs to find it first. Then, given the SDF, it is possible to solve for the market price-dividend ratio and the valuation ratios of other claims. The following proposition gives $z_{c,t}$ as a function of the state variables and model parameters.

Proposition 1. *The log price-consumption ratio $z_{c,t}$ is affine in the state variables,*

$$z_{c,t} = A_{c,0} + A_{c,1}x_t + A_{c,2}y_t + A_{c,3}\sigma_t^2 \tag{8}$$

and

$$\begin{aligned}
A_{c,2} &= \left(1-\frac{1}{\psi}\right)\frac{\phi_c}{1-\kappa_{c,1}(1+\phi_d-\phi_c)},\\
A_{c,1} &= \frac{1}{1-\kappa_{c,1}\rho}\left(1-\frac{1}{\psi}\right)\left[\psi_c+\kappa_{c,1}s(\psi_d-\psi_c)\right],\\
A_{c,3} &= \frac{1}{2}(1-\gamma)\left(1-\tfrac{1}{\psi}\right)\left[\frac{1+2\kappa_{c,1}s(\varphi\alpha-1)+(\varphi^2-2\alpha\varphi+1)(\kappa_{c,1}s)^2+(\varphi_x\kappa_{c,1}p)^2}{1-\kappa_{c,1}\nu}\right],\\
A_{c,0} &= \frac{1}{1-\kappa_{c,1}}\left[log(\delta)+(1-\tfrac{1}{\psi})\mu_c+\kappa_{c,0}+\kappa_{c,1}A_{c,2}(\mu_d-\mu_c)+\kappa_{c,1}A_{c,3}(1-\nu)\sigma^2+\tfrac{1}{2}(\theta\kappa_{c,1}A_{c,3}\sigma_w)^2\right],\\
s &\equiv \frac{\phi_c}{1-\kappa_{c,1}(1+\phi_d-\phi_c)},\\
p &\equiv \frac{\psi_c+\kappa_{c,1}s(\psi_d-\psi_c)}{1-\kappa_{c,1}\rho}.
\end{aligned}$$

A short outline of the derivation can be found in the appendix. It is important to note how the parameter ψ, that measures the EIS, affects the price-consumption ratio. Assume that $\gamma > 1$ and $\phi_c > 0$ (when dividends are above consumption, its growth is likely to accelerate). When $\psi < 1$, the expressions for $A_{c,1}$ and $A_{c,2}$ are negative, while the one for $A_{c,3}$ is positive. Intuitively, this would mean that the consumer values her consumption *more*, when expected consumption growth is *low*, the deviation from the cointegration relation is *low* (dividends below consumption) or the conditional volatility of consumption is *high*. The explanation is straightforward – for low EIS, when the consumer expects high growth in future consumption (either as an effect of higher conditional growth rate or dividends being high above consumption and pulling it up in expectations), she wants to borrow from the future to smooth her path of utility. This raises the interest rate to the extent such that the higher future consumption is heavily discounted, with negative effect on wealth. Similarly, when the volatility of consumption growth is higher, she is willing to save more because of precautionary motive, which decreases the interest rate and has positive effect on wealth. However, with EIS close to $1/\gamma$, the utility function is more similar to the power utility, and the model is not able to replicate high equity premium. This implies that in the long-run risks model of Bansal and Yaron [14] (and its variations), one needs EIS above one. Whether this is the case in the data is still an open question.[15]

The log price-consumption ratio in Proposition 1. depends on the log-linearization constants $\kappa_{c,0}$ and $\kappa_{c,1}$. Since they depend on the mean price-consumption ratio, every parameter change that influences the mean $z_{c,t}$ will also impact the kappas. I follow the

[14]Note that the word "price" in the price-dividend ratio and the price-consumption ratio actually refers to two different prices. I use these terms hoping that no confusion will arise.

[15]There have been many attempts to estimate the EIS. Hansen and Singleton [42], Vissing-Jorgensen [55], Vissing-Jorgensen and Attanasio [56] and Guvenen [39] report values above one. Hall [40] and Campbell [20] argue for values much closer to zero.

long-run risks literature and compute them endogenously for every set of parameters. The details are in the appendix.

3.3. **Stochastic discount factor and the risk-free rate.** Having solved for the price-consumption ratio, I can characterize the stochastic discount factor for the economy. I give the essentials in the proposition.

Proposition 2. *The log stochastic discount factor takes the form*

$$m_{t+1} = -m_0 - m_1 x_t - m_2 y_t - m_3 \sigma_t^2 - \Lambda_\eta \sigma_t \eta_{t+1} - \Lambda_u \sigma_t u_{t+1} - \Lambda_\varepsilon \sigma_t \varepsilon_{t+1} - \Lambda_w \sigma_w w_{t+1}, \tag{9}$$

with the coefficients and prices of risk given by

$$\begin{aligned}
m_0 &= -\theta log(\delta) + \gamma\mu_c + (1-\theta)\{\kappa_{c,0} + \kappa_{c,1}[A_{c,0} + A_{c,2}(\mu_d - \mu_c) + A_{c,3}(1-\nu)\sigma^2)] - A_{c,0}\},\\
m_1 &= \gamma\psi_c + (1-\theta)[\kappa_{c,1}(A_{c,1}\rho + A_{c,2}(\psi_d - \psi_c) - A_{c,1})],\\
m_2 &= \gamma\phi_c + (1-\theta)A_{c,2}[\kappa_{c,1}(1 + \phi_d - \phi c) - 1],\\
m_3 &= (1-\theta)A_{c,3}(\kappa_{c,1}\nu - 1);\\
\Lambda_\eta &= \gamma - (1-\theta)\kappa_{c,1}A_{c,2},\\
\Lambda_u &= (1-\theta)\kappa_{c,1}A_{c,2}\varphi,\\
\Lambda_\varepsilon &= (1-\theta)\kappa_{c,1}A_{c,1}\varphi_x,\\
\Lambda_w &= (1-\theta)\kappa_{c,1}A_{c,3}.
\end{aligned}$$

The appendix contains a brief outline of the derivation. The parameters $\Lambda_x,\ x \in \{\ \eta, u, \varepsilon, w\ \}$ measure the representative agent's risk aversion associated with the four shocks. With $\psi > 1$, we have $1 - \theta > 0$. The price of direct consumption risk (η_{t+1}) is equal to the coefficient of relative risk aversion, decreased by the amount to which the cointegrating relation hedges against shocks to the current consumption (if there is strong consumption growth, it is likely to partly revert in the future). The price of shocks to the dividend (u_{t+1}) is non-zero only because of the presence of cointegration – if $A_{c,2} = 0$, this price is zero as well. The shocks to the persistent conditional growth rate (ε_{t+1}) and conditional volatility (w_{t+1}) have positive prices of risk. Note that in this model, as in Bansal and Yaron [14], the prices of risk are constant.

What makes the market risk premia time-varying in the model, is the *amount* of risk given by σ_t. This is one of the most important differences between the long-run risks models and the models with habit formation,[16] in which consumption process is assumed to be homoskedastic and the *market price of risk* varies.

From (9) it is straightforward to obtain the (logarithmic) risk-free rate r_t^f, through

$$e^{-r_t^f} = E_t\left[e^{m_{t+1}}\right]. \tag{10}$$

The solution can be summarized as follows:

Proposition 3. *The continuously compounded risk-free rate is*

$$r_{t+1}^f = r_0^f + r_1^f x_t + r_2^f y_t + r_3^f \sigma_2, \tag{11}$$

$$\begin{aligned}
r_0^f &= m_0 - \frac{1}{2}\Lambda_w^2\sigma_w^2,\\
r_1^f &= m_1,\\
r_2^f &= m_2,\\
r_3^f &= m_3 - \frac{1}{2}\Lambda_\eta^2 - \frac{1}{2}\Lambda_u^2 - \Lambda_\eta\Lambda_u\alpha - \frac{1}{2}\Lambda_\varepsilon^2.
\end{aligned}$$

[16]For example, Campbell and Cochrane [21], or Wachter [54].

3.4. **Price-dividend ratio and market return.** To price the market portfolio (the claim to all future dividends), one needs the log-linearized market return (4) and the price-dividend ratio $z_{d,t}$. The following proposition shows its functional form.

Proposition 4. *The log price-dividend ratio $z_{d,t}$ is affine in the state variables,*

$$z_{d,t} = A_{d,0} + A_{d,1}x_t + A_{d,2}y_t + A_{d,3}\sigma_t^2, \tag{12}$$

with the coefficients given by

$$\begin{aligned}
A_{d,2} &= \frac{\phi_d - m_2}{1-\kappa_{d,1}(1+\phi_d-\phi_c)},\\
A_{d,1} &= \frac{\psi_d + \kappa_{d,1}A_{d,2}(\psi_d-\psi_c) - m_1}{1-\kappa_{d,1}\rho},\\
A_{d,3} &= \frac{1}{1-\kappa_{d,1}\nu}\left[\tfrac{1}{2}(\Lambda_\eta+\kappa_{d,1}A_{d,2})^2+\tfrac{1}{2}(\Lambda_u-\varphi(\kappa_{d,1}A_{d,2}+1))^2+\tfrac{1}{2}(\Lambda_\varepsilon-\kappa_{d,1}A_{d,1}\varphi_x)^2\right]\\
&\quad+\frac{1}{1-\kappa_{d,1}\nu}\left[(\Lambda_\eta+\kappa_{d,1}A_{d,2})(\Lambda_u-\varphi(\kappa_{d,1}A_{d,2}+1))\alpha - m_3\right],\\
A_{d,0} &= \frac{1}{1-\kappa_{d,1}}\left[\kappa_{d,0}+\mu_d+\kappa_{d,1}A_{d,2}(\mu_d-\mu_c)+\kappa_{d,1}A_{d,3}(1-\nu)\sigma^2\right]\\
&\quad+\frac{1}{1-\kappa_{d,1}}\left[\frac{1}{2}\sigma_w^2(\Lambda_w-\kappa_{d,1}A_{d,3})^2-m_0\right].
\end{aligned}$$

The proof that (12) really satisfies the no-arbitrage condition when plugged into the expression for the market return (4) is almost identical to that of Proposition 1, and I refer the reader to the appendix. Also in the appendix, I give the algorithm that can be applied to identify $\kappa_{d,0}$ and $\kappa_{d,1}$.

Define $r^m_{t+1} = log(D_{t+1}+P_{t+1})/P_t$ to be the logarithmic market return. The following proposition characterizes it in terms of the four underlying shocks.

Proposition 5. *The continuously compounded return on the aggregate market is*

$$r^m_{t+1} = r^m_0 + r^m_1 x_t + r^m_2 y_t + r^m_3\sigma_2 - R^m_\eta\sigma_t\eta_{t+1} - R^m_u\sigma_t u_{t+1} - R^m_\varepsilon\sigma_t\varepsilon_{t+1} - R^m_w\sigma_w w_{t+1}, \tag{13}$$

and the coefficients are

$$\begin{aligned}
r^m_0 &= \kappa_{d,0}+\mu_d - A_{d,0} + \kappa_{d,1}\left[A_{d,0}+A_{d,2}(\mu_d-\mu_c)+A_{d,3}(1-\nu)\sigma^2\right],\\
r^m_1 &= \psi_d - A_{d,1} + \kappa_{d,1}\left[A_{d,1}\rho + A_{d,2}(\psi_d-\psi_c)\right],\\
r^m_2 &= \phi_d - A_{d,2}(1-\kappa_{d,1}(1+\phi_d-\phi_c)),\\
r^m_3 &= -A_{d,3}(1-\kappa_{d,1}\nu);\\
R^m_\eta &= \kappa_{d,1}A_{d,2},\\
R^m_u &= -\varphi(1+\kappa_{d,1}A_{d,2}),\\
R^m_\varepsilon &= -\kappa_{d,1}A_{d,1}\varphi_x,\\
R^m_w &= -\kappa_{d,1}A_{d,3}.
\end{aligned}$$

3.5. **Term structure of equity.** The focus of this paper is to analyze risk properties of assets that pay their dividends at various maturities. I therefore decompose the aggregate price-dividend ratio into the infinite sum of price-dividend ratios for equity claims (*equity strips*)[17] that pay the aggregate dividend in the economy at fixed future time points. Define P^n_t/D_t as the price of aggregate dividend paid by the economy at date t+n, normalized by currently paid dividend. It is straightforward to observe, that

$$\frac{P_t}{D_t} = \sum_{n=1}^{\infty}\frac{P^n_t}{D_t}.$$

Define also R^n_{t+1} to be the gross return on an equity strip of maturity n. For $n = 1$, this return is only composed of the dividend payout at t+1, while for $n > 1$ there are only

[17] In this nomenclature, I follow Drechsler [30]. Similar decomposition of equity is also applied in Lettau and Wachter [47], [48] and Croce, Lettau and Ludvigson [27], among others.

capital gains, as in the case of long-maturity bonds. The return can be written

$$R^n_{t+1} = \frac{P^{n-1}_{t+1}}{P^n_t} = \frac{\frac{P^{n-1}_{t+1}}{D_{t+1}} D_{t+1}}{\frac{P^n_t}{D_t} D_t} = \frac{\frac{P^{n-1}_{t+1}}{D_{t+1}}}{\frac{P^n_t}{D_t}} \frac{D_{t+1}}{D_t}.$$

Taking logarithms,

$$r^n_{t+1} \equiv log(R^n_{t+1}) = log(\frac{P^{n-1}_{t+1}}{D_{t+1}}) - log(\frac{P^n_t}{D_t}) + \Delta d_{t+1}. \tag{14}$$

I now conjecture the functional form of $log(P^n_t/D_t)$ to be

$$log(\frac{P^n_t}{D_t}) = Z_{0,n} + Z_{1,n}x_t + Z_{2,n}y_t + Z_{3,n}\sigma^2_t,$$

which allows me to rewrite (14) as

$$\begin{aligned} r^n_{t+1} &= \left[Z_{0,n-1} + Z_{1,n-1}x_{t+1} + Z_{2,n-1}y_{t+1} + Z_{3,n-1}\sigma^2_{t+1}\right] \\ &\quad - \left[Z_{0,n} + Z_{1,n}x_t + Z_{2,n}y_t + Z_{3,n}\sigma^2_t\right] + \Delta d_{t+1}. \end{aligned} \tag{15}$$

It is now straightforward to use the dynamics of the state variables (3) and the no-arbitrage condition (2) applied to (17), to derive a set of Riccati difference equations relating the coefficients $Z_{i,n}$ to $Z_{i,n-1}$; $i \in \{0,..,3\}$:

$$\begin{aligned} Z_{0,n} &= \mu_d - m_0 + Z_{0,n-1} + Z_{2,n-1}(\mu_d - \mu_c) + Z_{3,n-1}(1-\nu)\sigma^2 \\ &\quad + \frac{1}{2}(\Lambda_w - Z_{3,n-1})^2\sigma^2_w, \\ Z_{1,n} &= \psi_d - m_1 + Z_{1,n-1}\rho + Z_{2,n-1}(\psi_d - \psi_c), \\ Z_{2,n} &= \phi_d - m_2 + Z_{2,n-1}(1 + \phi_d - \phi_c), \\ Z_{3,n} &= -m_3 + Z_{3,n-1}\nu + \frac{1}{2}(\Lambda_\eta + Z_{2,n-1})^2 + \frac{1}{2}(\Lambda_u - \varphi(Z_{2,n-1}+1))^2 \\ &\quad + \frac{1}{2}(\Lambda_\varepsilon - Z_{1,n-1}\varphi_x)^2 + (\Lambda_\eta + Z_{2,n-1})(\Lambda_u - \varphi(Z_{2,n-1}+1))\alpha. \end{aligned} \tag{16}$$

The system can be solved recursively, using the set of the initial conditions $Z_{0,0} = Z_{1,0} = Z_{2,0} = Z_{3,0} = 0$, which guarantee that the price of the dividend at time t is exactly D_t (known at t), so that $P^0_t/D_t = 1$.[18]

Knowing the valuation ratios of all equity strips, it is useful for further analysis of their excess returns to write down the returns as functions of the shocks to the economy. The proposition below follows from (15) and (16), together with (3).

Proposition 6. *The continuously compounded return on an equity strip that gives the right to the aggregate dividend at time t+n is*

$$r^n_{t+1} = r^n_0 + r^n_1 x_t + r^n_2 y_t + r^n_3 \sigma_2 - R^n_\eta \sigma_t \eta_{t+1} - R^n_u \sigma_t u_{t+1} - R^n_\varepsilon \sigma_t \varepsilon_{t+1} - R^n_w \sigma_w w_{t+1}, \tag{17}$$

where the coefficients are

$$\begin{aligned} r^n_0 &= \mu_d - Z_{0,n} + Z_{0,n-1} + Z_{2,n-1}(\mu_d - \mu_c) + Z_{3,n-1}(1-\nu)\sigma^2, \\ r^n_1 &= \psi_d - Z_{1,n} + Z_{1,n-1}\rho + Z_{2,n-1}(\psi_d - \psi_c), \\ r^n_2 &= \phi_d - Z_{2,n} + Z_{2,n-1}(1 + \phi_d - \phi_c), \\ r^n_3 &= -Z_{3,n} + Z_{3,n-1}\nu; \\ R^n_\eta &= Z_{2,n-1}, \\ R^n_u &= -\varphi(Z_{2,n-1}+1), \\ R^n_\varepsilon &= -Z_{1,n-1}\varphi_x, \\ R^n_w &= -Z_{3,n-1}. \end{aligned}$$

[18]Since the focus of this paper is on equity, I do not solve for the zero-coupon bond prices. This is possible using the same set of difference equations, but assuming constant payouts of one at every maturity, instead of the aggregate dividends.

4. Calibration

The model operates at monthly frequency. However, following the standard approach in the literature, I calibrate it to the moments of yearly data,[19] aggregating the monthly series. The parameters are summarized in Table 1, which also shows the calibrations of Bansal and Yaron (BY) [14] and Bansal, Kiku and Yaron (BKY) [9], [10], which are usually considered as benchmarks for the long-run risks models.

4.1. **Parameter values.** The preference parameters $\gamma = 15$ and $\psi = 1.5$ are the same for all three calibrations. I chose the time preference parameter $\delta = 0.9989$ – the same as in BKY. The parameters governing the consumption process are identical to those in BY. In particular, I restrict the consumption-related coefficient on the deviation from the cointegration relation, ϕ_c to be zero, which is a simplifying assumption and means that in the long run consumption provides a trend for dividends.[20] The properties of the persistent component of consumption growth x_t are the same as in BY. $\rho = 979$ implies a half life of shocks to x_t of about 2.7 years.

The calibration of the dividend process is slightly different from both BY and BKY, although closer to the latter. I choose the "leverage" parameter at $\phi_d = 2$, which means that long-run component x_t affects expected dividend growth two times stronger than consumption growth.[21] I slightly raise the parameter φ that governs conditional volatility of dividend growth, from 6.5 of BKY to 8.0, and set the correlation coefficient between consumption and dividend growth to $\alpha = 0.5$, compared to 0.4 in BKY.

The cointegration relation in my model operates through dividends. I set $\phi_d = -0.001$, implying that any deviation from the cointegration is very persistent.[22] This makes the model seemingly similar to the one without cointegration, but as I show below, it is needed to capture the pattern of asset betas needed for the main result. Intuitively, since the long-run risk component that produces the positively-sloped pattern of betas is very small, a small impact of cointegration is enough to undo its effect.

Finally, the set of parameters governing the conditional variance process is identical to that of BY. In the calibration of BKY, the mean σ_2 is slightly lower, while its conditional volatility σ_w, higher. Coupled with much higher persistence parameter ν, these assumptions imply much higher unconditional variability of σ_t^2 around the (slightly lower) unconditional mean. As pointed out by Beeler and Campbell [16], this implies higher probability that the variance process hits zero at some point.[23] In the calibration of BY, such events are virtually absent. Moreover, extreme persistence of consumption volatility implied by BKY (the half-life of shocks of 58 years) seems to be exaggerated.

4.2. **Calibration results.** The set of moments to be matched is standard and includes the dynamics of consumption growth, dividend growth, market excess return, the risk-free rate and market price-dividend ratio. For each of them, I consider the mean growth rate, standard deviation of the growth rate and the first-order autocorrelation coefficient. Table 2. shows that the model does a decent job in matching these moments.

The mean consumption growth and its autocorrelation are close to the data. The only exception is consumption growth volatility, which seems to be too high. Bansal, Kiku and Yaron [10] argue that the value in the data is within 95% confidence interval around their model's population value, which implies that the latter is not too unusual to be rejected. Since my model performs better than BKY in matching this parameter, the same statement must be true here as well.

The moments of the dividend growth are noticeably different from the empirical counterparts – especially the volatility and autocorrelation seem to be too high. However, the moments are better fitted to the data if one compares them to those of the *earnings*. For

[19]See Kandel and Stambaugh [43], Campbell and Cochrane [21] or Bansal and Yaron [14].

[20]This is consistent with the evidence that consumption growth is virtually unforecastable by variables other than its lagged values. See Cochrane [26].

[21]I argue that this parameter should not be too far from one – if there was a dividend smoothing firm in the model, the optimal policy would be to set expected dividend growth at expected consumption growth at every point in time. There is evidence that firms actually smooth dividends, especially in the post-war period. See Chen, Da and Priestley [25].

[22]The autoregressive coefficient of the deviation from the cointegration relation is 0.999.

[23]In such a case it is replaced with a small positive number.

the sample between 1930 and 2006, average earnings growth was 2.11%, their volatility amounted to 16.97% and autocorrelation equaled 0.29. These numbers are much closer to the model-implied population values.[24]

The equity premium, its volatility, and autocorrelation are matched by the model very well. This is no surprise, given that the model of Bansal and Yaron [14] was designed to resolve the equity premium puzzle. The moments of the real interest rate are different than in the data, but paradoxically this can be considered a strength of the model[25] – the high volatility of the real interest rate in the real world is due to frequent changes in inflation. This also explains the relatively low persistence of the empirical value.

Finally, the model matches the moments of the price-dividend ratio relatively well, better than the calibrations of BY and BKY. It slightly understates the volatility, but this moment is known to be difficult to match by the long-run risks models.[26]

5. The Results

5.1. **The effects of payout horizon.** I first analyze the effect of payout horizon on the riskiness of individual equity strips. Using the law of iterated expectations, the unconditional expected gross return of the n-th strip is

$$E\left(R_{t+1}^{n}\right) = E\left(e^{r_{t+1}^{n}}\right) = E\left(E_t(e^{r_{t+1}^{n}})\right) = E\left(e^{E_t(r_{t+1}^{n}) + \frac{1}{2}Var_t(r_{t+1}^{n})}\right). \tag{18}$$

Substituting (17) for r_{t+1}^{n} and computing the conditional expectation, I can re-write the result as

$$E\left(R_{t+1}^{n}\right) = E\left[e^{\left[r_0^n + \frac{1}{2}R_w^{n2}\right] + r_1^n x_t + r_2^n y_t + \left[r_3^n + \frac{1}{2}\left(R_\eta^{n2} + R_u^{n2} + 2R_\eta^n R_u^n \alpha + R_\varepsilon^{n2}\right)\right]\sigma_t^2}\right].$$

It remains now to calculate the unconditional expected value, which is

$$E\left(R_{t+1}^{n}\right) = e^{\gamma_0 + \gamma_2 \bar{y} + \gamma_3 \sigma^2 + \frac{1}{2}\left(\gamma_1^2 c_{xx} + \gamma_2^2 c_{yy} + 2\gamma_1\gamma_2 c_{xy} + \gamma_3^2 \varsigma\right)}; \tag{19}$$

$$\begin{aligned}
\gamma_0 &= r_0^n + \frac{1}{2}\left(R_w^n\right)^2, \\
\gamma_1 &= r_1^n, \\
\gamma_2 &= r_2^n, \\
\gamma_3 &= r_3^n + \frac{1}{2}\left(\left(R_\eta^n\right)^2 + \left(R_u^n\right)^2 + 2R_\eta^n R_u^n \alpha + \left(R_\varepsilon^n\right)^2\right),
\end{aligned}$$

where c_{xx}, c_{xy}, c_{yy} are unconditional variances and covariances of state variables x_t and y_t; ς is the unconditional variance of σ_t^2; and $\bar{y}$ is the unconditional mean of y_t.[27]

The unconditional expectations of the risk-free rate and the return on aggregate market, which I denote $E(R^f)$ and $E(R^m)$ respectively, can be obtained in an analogous way, using formulas (11) and (13). The unconditional excess expected return of the strips is then $E(R^n - R^f)$, and the expected excess return on the market is $E(R^m - R^f)$.

Figure 1 compares the unconditional excess returns earned in expectations by the equity strips with various maturities (solid line) with expected excess return on the market (dashed line), after scaling them by factor 12 to convert into yearly values. Excess returns on equity strips firs increase, to reach the maximum value of 6.47% at maturity of around 150 months (12.5 years), and then decrease steadily. The strip with the shortest maturity of 1 month earns only 3.01% above the risk-free rate, so the difference between the highest and lowest return in the figure is almost 3.5 percentage points.

Equity strips with the shortest maturities earn low excess returns, because their prices covary only weakly with the innovations to the "long-run risk" state variables x_t and σ_t^2.[28] The aggregate dividend paid in the first month (the shortest equity strip) is not affected at all by innovations to expected consumption growth and variance, since the latter have

[24]The data on dividends and earnings can be downloaded on Robert Shiller's website.

[25]This point was made by Beeler and Campbell [16].

[26]See Beeler and Campbell [16]. Bansal and Shaliastovich [12] made some progress in this respect, by introducing a third channel for asset price variability – time-varying confidence risk. In their model, the long run component is unobservable by the representative agent, who only observes a set of signals. The precision of the signals (the *confidence*) is time-varying.

[27]Note that $\bar{x} = 0$, so it does not appear in (19). Similarly, $\bar{\sigma}^2$ – the long run mean of σ_t^2, is just σ^2.

[28]Asset prices' covariation with innovations to y_t does not play a direct role here, because in my calibration consumption growth is unaffected by y_t.

effect only on payouts starting from month t+2 on. As maturity grows, long-run risks have more and more impact on equity, depressing the prices and increasing excess returns. However, for very long maturities, even the effect of persistent growth and variance are gradually wiped out by the cointegrating relation between dividend and consumption *levels*. Looking at (3), we can see that both x_t and σ_t^2 affect the dividend process more than consumption, through "leverage" parameters ψ_d and φ, which are both greater than one. But because of the presence of cointegration, the expected dynamics of dividends cannot be different in the very long run from the dynamics of consumption.

The natural question that arises is whether the low returns of short-maturity equity strips can be explained by the covariation with the market return, or equivalently – whether the CAPM works well within the model. I start with the definition of the unconditional market beta for an n-th equity strip,

$$\beta_m^b = \frac{Cov(R^n, R^m)}{Var(R^m)}. \tag{20}$$

Below, I rewrite the gross returns in terms of exponential (continuously compounded) returns. Using the law of total covariance for the numerator and the law of total variance for the denominator, the formula becomes

$$\beta_m^b = \frac{E\left[Cov_t\left(e^{r_{t+1}^n}, e^{r_{t+1}^m}\right)\right] + Cov\left[E_t\left(e^{r_{t+1}^n}\right), E_t\left(e^{r_{t+1}^m}\right)\right]}{E\left[Var_t\left(e^{r_{t+1}^m}\right)\right] + Var\left[E_t\left(e^{r_{t+1}^m}\right)\right]}. \tag{21}$$

(21) can be solved in a closed-form, although the algebra is a bit tedious.

Figure 2 graphs the unconditional market betas of the equity strips as a function of maturity. It can be seen that under my calibration, the CAPM can potentially work well for longer maturities, as decreasing betas fit the pattern of decreasing expected returns. This is, however, not the case for the short maturities, where expected returns are *increasing* function of maturity, which obviously cannot be explained by the pattern of betas.[29] It is interesting to note that the betas actually increase for the extremely short maturities, which however cannot turn the previous conclusion around.[30] Moreover, this pattern is roughly in line with the empirical results that I discuss below.

Why are the betas decreasing in maturity? To answer this question, I decompose conditional market betas of the equity strips into parts that come from covariation of market return with sources of variation in individual strips, assuming that the state variables are at their long-run mean values.[31] The use of *conditional*, rather than *unconditional* betas is for simplicity of illustration, the decomposition of the latter results in exactly the same intuition. For a similar reason, I use logarithmic returns. The conditional beta is

$$\beta_{m,t}^i = \frac{Cov_t(r_{t+1}^n, r_{t+1}^m)}{Var_t(r_{t+1}^m)}.$$

Using (17), it can be re-written as

$$\begin{aligned}\beta_t^n &= \frac{Cov_t(-R_\eta^n\sigma_t\eta_{t+1} - R_u^n\sigma_t u_{t+1} - R_\varepsilon^n\sigma_t\varepsilon_{t+1} - R_w^n\sigma_w w_{t+1}, r_{t+1}^m)}{Var_t(r_{t+1}^m)} \\ &\equiv \beta_{\eta,t}^n + \beta_{u,t}^n + \beta_{\varepsilon,t}^n + \beta_{w,t}^n.\end{aligned}$$

The resulting components are plotted in Figure 3. The main part of the betas of individual strips comes from their covariation with u_{t+1} – the shock to dividend growth (bold, dashed line). This component is also responsible for the decreasing pattern of betas. The portion that comes from the covariation with the innovation to x_t, the long-run risk variable, is smaller and it increases with maturity for the shortest strips (bold, dotted line). The effects from direct exposure to innovations in consumption growth η_{t+1} (thin, solid

[29]This result is in contrast with the findings of Drechsler [30], who uses a similar model, under calibrations different from mine. He does not concentrate on the shortest-maturity equity and concludes that the CAPM preforms well.

[30]It is actually possible to calibrate the model to obtain an exactly decreasing pattern of CAPM betas, by slightly decreasing the parameter ψ_d. Another effect of doing this would be marginally smaller market equity premium.

[31]Actually, only σ_t^2 matters for conditional covariances.

line) and conditional volatility σ_t^2 (thin, dotted line) are much smaller.[32] To highlight the importance of the cointegration, I also plot analogous decomposition within a model in which I assume it away. The results are in Figure 4, and it is clear that now the pattern of betas is that they increase in maturity, especially for the shorter end of the maturity spectrum. Overall, the effect that comes from the cointegrating relation is solely responsible for decreasing pattern of the CAPM betas under my calibration.

5.2. **Implications for distressed equity.** I model distress in a reduced-form fashion, employing a set of simplifying assumptions. The firms are parametrized by a constant probability of default. Each firm pays a constant (possibly very small) portion of the aggregate dividend until it defaults. In the event of default at time t, the firm will pay the dividend at t+1, but will pay nothing in the following periods. Finally, I assume that default is caused by purely idiosyncratic reasons.[33]

I consider ten firms (which can also be interpreted as portfolios) with ten different *per year* default probabilities $p \in \{0.0011\%, 0.014\%, 0.018\%, 0.024\%, 0.036\%, 0.057\%, 0.109\%, 0.192\%, 0.340\%, 0.803\%\}$, which correspond exactly to the fitted probabilities of default for ten groups of firms in the empirical study of Campbell et al. [24]. The groups are based on percentile cutoffs: the first two are computed within the first and the second 5-percents of the healthiest firms. The third entry corresponds to the firms between 10th and 20th percentile, and the three following numbers – to groups with the size of 20% each. The seventh probability is the average for firms between 80% and 90% of the distribution. The last three numbers refer to the most distressed groups, of size 5%, 4% and 1%, respectively.

In figures 5 and 6, I replicate the results from figures 2 and 3 in Campbell at al., focusing on unconditional mean excess returns, CAPM betas and alphas. Figure 5 presents unconditional mean excess returns (solid line) and CAPM alphas (dotted line) for ten distress-sorted portfolios (the most distressed firms on the right). It is clear from this figure, that extremely low returns earned by the most distressed stocks provide a challenge for non-behavioral models of asset pricing. Importantly, CAPM alphas are even *below* unconditional returns, which means that the standard CAPM fails completely in describing the empirical pattern. Figure 6 provides a confirmation: the CAPM beta and the loading on Fama and French's MKT factor are *increasing* in distress.[34] These patterns are very difficult to replicate exactly within a risk-based model, because of the magnitude of the pricing errors. However, it turns out that my calibration of the long-run risks model can replicate them well at least *qualitatively.*

Under the assumptions above, the expected return that the firm earns from t to t+1 is summarized by the distribution of the random default time, and the riskiness of the dividend strips paid until the default occurs. Before characterizing expected excess returns and market betas, it is useful to start with a hypothetical firm with deterministic future time of default $d > t$. Its future payouts are composed of (constant) portions of aggregate equity strips of maturities between t+1 and d+1. Expected excess gross return R^d_{t+1} of this firm is the average of excess gross returns of the individual strips:

$$E(R^{d,c}_{t+1} - R^f_{t+1}) = \frac{1}{d} \sum_{k=1}^{d+1} E(R^k_{t+1} - R^f_{t+1}). \tag{22}$$

[32]This is because my calibration of long-run risk processes, especially their persistence, is close to the model of Bansal and Yaron [14], in which the main source of risk are shocks to x_t.

[33]I make this assumption for the sake of simplicity, to show the main point of the paper – *conditional* on being in distress, firms have short expected lifetimes, so that they *do not* load on the long-run risk factors. Even if these factors affect their future prospects conditional on no-default, purely idiosyncratic shocks to cash flows appear to be much more important to resolve the uncertainty of whether the firm defaults or not: A distressed firm has high leverage, difficulties in paying debt coupons, and prohibitively high costs of raising additional equity to keep paying its liabilities.

Note also, that the model considered here assumes away the effects of business cycles, which are usually associated with increased numbers of bankruptcies. Indeed, Bansal et al. [11] show that within the class of long-run risks models, business cycle risk remains only weakly priced by the representative agent, when the EIS is not very close to zero. My assumption does not in fact rule out defaults that are brought about by high-frequency movements in economic conditions, were they present in the model.

[34]Campbell et al. do not report CAPM betas. My dotted line is implied from their unconditional returns and CAPM alphas, together with the assumption of mean excess market return of 0.0575.

Equation (22) defines a term structure of what I name *cumulative equity strips.* Figure 7 compares the cumulative strip excess returns (solid line) to the returns of individual strips (dotted line).

Similarly, one can think of unconditional market betas of firms that default at a known time $d > t$. From (22), it follows immediately that this beta is a weighted average of market betas of individual strips that constitute this firm's future payouts:

$$\beta_m^{d,c} = \frac{1}{d}\sum_{k=1}^{d+1} \beta_m^k. \tag{23}$$

The difference between the cumulative and individual betas is illustrated in figure 8. The pattern of the former (solid line) is more smooth, and flatter.

The following proposition provides a way to compute expected excess returns on firms indexed by the default probability, and their market betas.

Proposition 7. *Consider a firm with constant default probability p per period. Let T be the associated (random) time of default, uncorrelated with the shocks to the state variables in the model.*

(1) *The expected excess return of the firm is*

$$E(R_{t+1}^p - R_{t+1}^f) = E(R_{t+1}^{T,c} - R_{t+1}^f).$$

(2) *The unconditional market beta of the firm is*

$$\beta_m^p = E(\beta_m^{T,c}).$$

The proposition says that both expected returns and betas of a firm with random default time can be computed by taking simple expectations of cumulative strip returns and cumulative betas, under the distribution implied by the time of default T. The proof is given in the appendix. The second part hinges upon the assumption that the time of default is uncorrelated with the state variables, and necessarily also with the market return.

It is now easy to compute the expected excess returns, betas and alphas for the ten firms of interest, with exactly the same probabilities of default as the ten groups in Campbell et al. The results are in figures 9 through 11. The model replicates the decreasing pattern of returns. Notably, the returns for the first five (the most healthy firms) are relatively flatter as a function of the measure of distress, than the other five – most distressed groups. This is a qualitative success of the model, since the empirical pattern shown in figure 5 is very similar to the model-implied one. *Quantitatively* though, the model is not able to reproduce the very large negative returns for the most distressed firms, which are an annualized minus 16% in the extreme case. Figure 11 shows that the same applies to CAPM alphas, which also decrease in the direction of distress, being relative flat at the beginning, and then dropping sharply to minus 3% in annual terms.

Similarly, the model replicates well the pattern of market betas (figure 10). They are generally increasing in maturity, falling slightly for the firms with the two highest probabilities of default. This is again very similar to the empirical pattern, reported in figure 6. Taking the results together, the conclusion is that under my calibration, the long-run risks model reproduces the failure of the CAPM in pricing of the distressed firms. Interestingly, the CAPM works quite well for longer-maturity equity, and the alphas associated with the healthiest firms are closer to zero.

6. Conclusions

This paper proposes a consumption-based explanation of the empirical fact that distressed firms earn low expected returns, despite having higher loadings on market risk. I employ a long-run risks framework similar to that of Bansal and Yaron [14], with additional assumption of cointegrated consumption and dividend processes. The firms with high default likelihood are necessarily those that have short expected maturity of cash flows – they pay the bulk of all their lifetime dividends in the very near future. Despite earning low returns, they have high market betas, because they covary more strongly with innovations to aggregate dividends: in the long run the innovations to dividends are wiped out by the slowly working effect of cointegration, which pulls dividends down to the long-run trend, defined by the level of consumption.

I model defaults in a reduced-form fashion, but the mechanism presented here must also be present for a fully-specified structural model of default (for example, Fisher, Heinkel and Zechner [34]), together with the long-run risks macroeconomic framework (Bhamra, Kuehn and Strebulaev [17]). To the extent that distressed firms have short expected lifetimes, they must have lower expected returns, *ceteris paribus*. It is a very interesting task for future research to see to what extent can this be reversed (or strengthened) by financial leverage, which of course depends on how are defaults related to macroeconomic conditions. Obviously, when the long-term prospects for economic growth are worsening, we should expect some events of default, but we should also expect defaults when there are radical technological *improvements*, which is the schumpeterian effect of creative destruction.

Appendix

The proof of Propositions 1. and 4. Assume that the functional form of the log price-consumption ratio is (8). The aim is to verify that the return to the consumption claim satisfies (2), when the coefficients $A_{c,i}$ (for $i \in \{0,..,3\}$) are set to the values in the proposition.

For notational convention, in the proof I suppress the subscripts c wherever they appear. Thus, for example, $r_{c,t+1}$ is just r_{t+1}.

The no-arbitrage condition for the consumption claim is

$$E_t[e^{m_{t+1}+r_{t+1}}] = 1.$$

For convenience, I rewrite the formulas for the log SDF (1) and the return on wealth (6), which appear in the exponent,

$$\begin{aligned} m_{t+1} &= \theta log(\delta) - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1)r_{t+1}, \\ r_{t+1} &= \kappa_0 + \Delta c_{t+1} + \kappa_1 z_{t+1} - z_t. \end{aligned}$$

The sum of the two is therefore

$$m_{t+1} + r_{t+1} = \theta log(\delta) + (1-\frac{1}{\psi})\Delta c_{t+1} + \theta(\kappa_0 + \kappa_1 z_{t+1} - z_t).$$

The following steps are a bit tedious, but straightforward. The expressions for z_{t+1} and z_t can be replaced using the conjectured function (8). Now, the sum is expressed only in terms of the dynamics of consumption plus the state variables at times t and $t+1$. One can use the specification (3) to write everything in terms of state variables x_t, y_t, σ_t^2 and four normal shocks $(\eta, u, \varepsilon, w)$. Now it can be seen that the exponent is a normal random variable. Taking the conditional expectation and noting that the coefficients at all three state variables plus the constant term must all be zero to ensure no-arbitrage, one obtains four equations that can be solved for the four coefficients $A_{c,i}$ in Proposition 1. An identical order of steps applies to the price-dividend ratio formula (12).

The constants of log-linearization. The constants $\kappa_{c,0}$ and $\kappa_{c,1}$ in (8) can be obtained with the help of an iterative search for the fixed point of $\bar{z}_c$. The algorithm is summarized as follows:

(1) Start with an initial guess of $\bar{z}_c$.
(2) Compute $\kappa_{c,0}$ and $\kappa_{c,1}$ using (7).
(3) Calculate the coefficients $A_{c,i}$ in Proposition 1.
(4) Obtain a new value of $\bar{z}_c$, using[35]

$$\bar{z}_c = A_{c,0} + A_{c,1}\bar{x} + A_{c,2}\bar{y} + A_{c,3}\sigma^2 = A_{c,0} + A_{c,2}\frac{\mu_d - \mu_c}{\phi_c - \phi_d} + A_{c,3}\sigma^2.$$

(5) With new value of $\bar{z}_c$, repeat steps (2)–(4) until convergence.

[35] Note that the long-run mean of x_t is zero. When the mean growth rates of consumption and dividends are equal, then also the unconditional mean of y_t is zero. If the model is restricted in a way that the cointegration relation has no impact on consumption and dividend growth rates ($\phi_c = \phi_d = 0$), the formula has to be modified to $\bar{z}_c = A_{c,0} + A_{c,3}\sigma^2$.

This algorithm converges very quickly (usually 10-20 iterations). The same method can be used to find $\kappa_{d,0}$ and $\kappa_{d,1}$.

The proof of Proposition 7. For the part one, note that $E(R^{d,c}_{t+1} - R^f_{t+1})$ is the expected return *conditional* on $T = d$. Taking *unconditional* expectation with respect to the distribution of T yields the result.

For the second part, I use the definition of market beta,

$$\beta^p_m = \frac{Cov(R^p_{t+1}, R^m_{t+1})}{Var(R^m_{t+1})}, \tag{24}$$

where R^p_{t+1} is the realized return on the firm with probability p per period of time, which induces a random time of default T. Without loss of generality, assume that the time of default can take only two values, $T \in$ {s,l} (short, long), with associated probabilities p_l and p_s. Define $E_T(.)$, $Cov_T(.)$ and $Var_T(.)$ as operators conditional on the information about the time of default. Equation (24) can be re-written as

$$\frac{Cov(R^p_{t+1}, R^m_{t+1})}{Var(R^m_{t+1})} = \frac{E[Cov_T(R^p_{t+1}, R^m_{t+1})] + Cov(E_T(R^p_{t+1}), E_T(R^m_{t+1}))}{Var(R^m_{t+1})},$$

where I used the law of total covariance for the numerator. The second term in the sum is zero, because $E_T(R^m_{t+1})$ is a constant, equal just to the expected return on the market (I am using the assumption that T is uncorrelated with the market). One can write further the numerator as

$$E[Cov_T(R^p_{t+1}, R^m_{t+1})] = p_s Cov_s(R^p_{t+1}, R^m_{t+1}) + p_l Cov_l(R^p_{t+1}, R^m_{t+1}), \tag{25}$$

with obvious notation. For the denominator, it is true that

$$Var(R^m_{t+1}) = Var_s(R^m_{t+1}) = Var_l(R^m_{t+1}),$$

which, together with (25), leads to

$$\beta^p_m = p_s \frac{Cov_s(R^p_{t+1}, R^m_{t+1})}{Var_s(R^m_{t+1})} + p_l \frac{Cov_l(R^p_{t+1}, R^m_{t+1})}{Var_l(R^m_{t+1})} = E(\beta^{T,c}_m).$$

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(A) Preference Parameters

	γ	ψ	δ		
me	10	1.5	.9989		
BY	10	1.5	.998		
BKY	10	1.5	.9989		

(B) Consumption Growth

	μ_c	ψ_c	ϕ_c	ρ	φ_e
me	.0015	1	0	.979	.044
BY	.0015	1	0	.979	.044
BKY	.0015	1	0	.975	.038

(C) Dividend Growth

	μ_d	ψ_d	ϕ_d	φ	α
me	.0015	2.0	-.001	8.0	.5
BY	.0015	3.0	0	4.5	.0
BKY	.0015	2.5	0	6.5*	.4*

(D) Volatility

	σ	σ_w	ν		
me	.0078	.0000023	.987		
BY	.0078	.0000023	.987		
BKY	.0072	.0000028	.999		

Table 1. Calibration of parameters. The first row in each panel corresponds to my calibration, the second to Bansal and Yaron [14] and the third to Bansal, Kiku and Yaron [10]. All parameters are monthly.

Model Implied Moments

Moment	me	-std	+std	BY	BKY	Data
$E(\Delta c)$	1.80	1.11	2.49	1.79	1.82	1.95
$\sigma(\Delta c)$	2.92	2.58	3.26	2.92	2.96	2.16
$\rho_1(\Delta c)$	0.52	0.41	0.63	0.51	0.44	0.44
$E(\Delta d)$	1.80	-0.21	3.81	1.66	1.85	1.02^a
$\sigma(\Delta d)$	18.00	16.22	19.78	11.57	16.42	10.69^b
$\rho_1(\Delta d)$	0.27	0.16	0.38	0.40	0.29	0.14^c
$E(r^m - r^f)$	5.75	3.54	7.96	6.62	6.58	6.20
$\sigma(r^m - r^f)$	18.60	16.64	20.56	16.88	21.35	18.34
$\rho_1(r^m - r^f)$	0.00	-0.12	0.12	0.03	0.02	0.04
$E(r^f)$	1.46	1.04	1.88	2.56	0.99	0.99
$\sigma(r^f)$	1.37	1.16	1.58	1.30	1.28	4.28
$\rho_1(r^f)$	0.78	0.70	0.86	0.85	0.86	0.59
$E(p - d)$	3.39	3.19	3.59	3.00	3.04	3.31
$\sigma(p - d)$	0.37	0.32	0.42	0.16	0.26	0.46
$\rho_1(p - d)$	0.91	0.77	1.05	0.77	0.95	0.88

Table 2. Model-implied moments. To compute the population moments, I run a hundred repetitions of 10000 years of simulated data (which gives a total of 12 m months), and average the model-implied moments across the repetitions. The last column in the table is taken from Beeler and Campbell [16], who use yearly time series for the time-span 1930-2006. For the fields with a-c superscripts, I calculate analogous moments using the data from 1930 to 2006 on real earnings from Robert Shiller's website. The results (for the same time span) are: a–2.11, b–16.97, c–0.29.

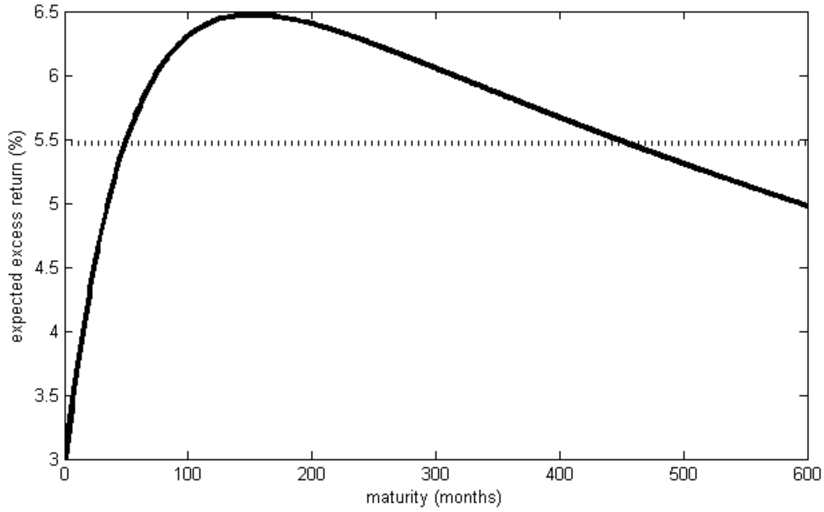

Figure 1. Expected excess returns of individual equity strips with various maturities (solid line), and expected excess return on the market (dotted line).

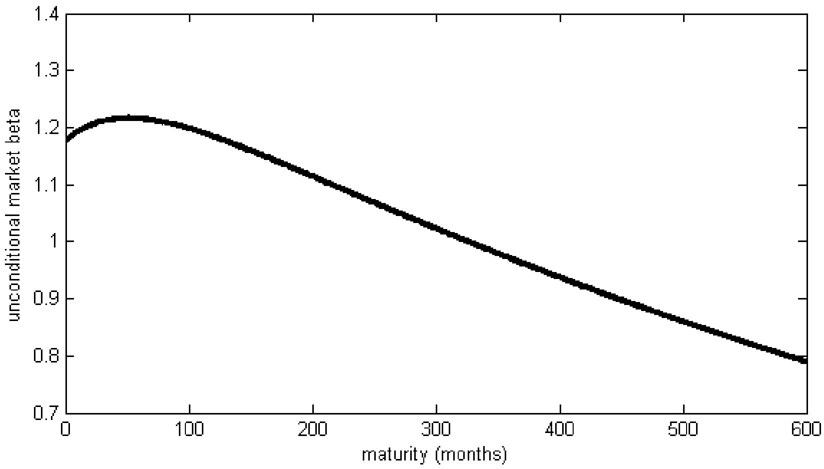

Figure 2. Unconditional (CAPM) betas of the equity strips with respect to the market portfolio.

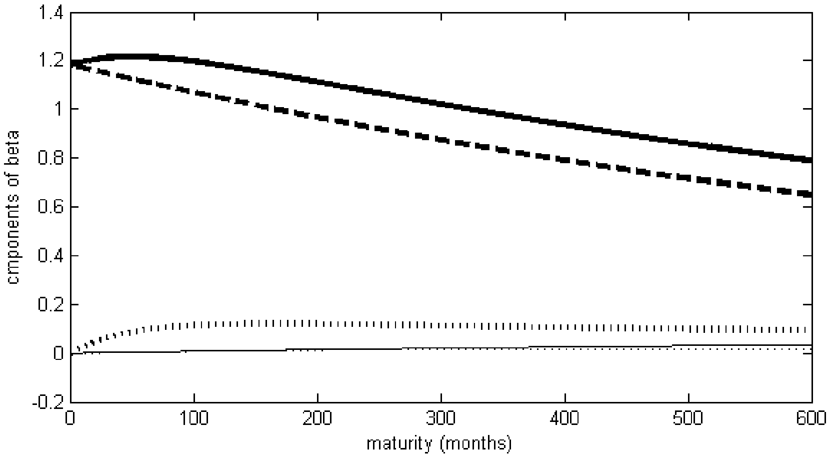

Figure 3. Decomposition of conditional market betas of individual equity strips (thick solid line) into portions due to the covariation of the strips with various risk factors: shocks to dividend (thick dashed line), shocks to the long-run risk component x_t (thick dotted line), shocks to consumption (thin solid line) and shocks to conditional variance of consumption growth σ_t^2 (thin dotted line). The parameter measuring the strength of cointegration is set to $\phi_d = -0.001$.

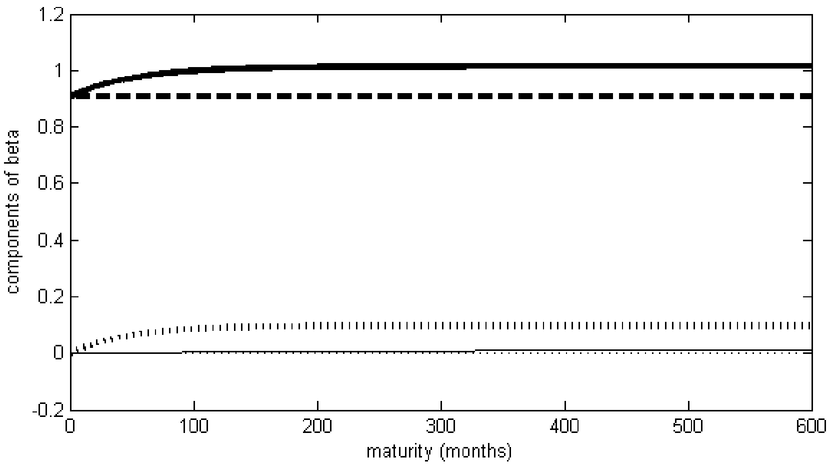

Figure 4. Decomposition of conditional market betas of individual equity strips (thick solid line) into portions due to the covariation of the strips with various risk factors: shocks to dividend (thick dashed line), shocks to the long-run risk component x_t (thick dotted line), shocks to consumption (thin solid line) and shocks to conditional variance of consumption growth σ_t^2 (thin dotted line). The parameter measuring the strength of cointegration is set to $\phi_d = 0$.

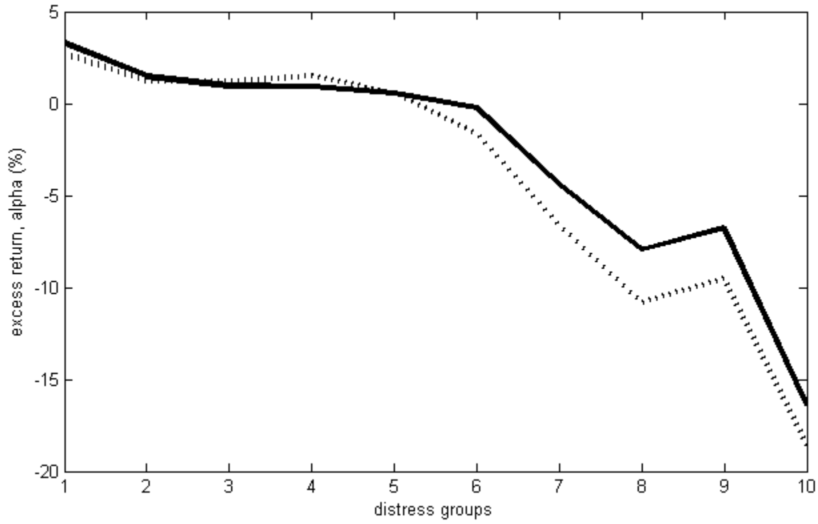

Figure 5. Mean excess returns for ten portfolios sorted with respect to *per year* probability of default (solid line), and CAPM alphas of these portfolios (dotted line). The probabilities of default are (from left to right): $p \in \{0.0011\%, 0.014\%, 0.018\%, 0.024\%, 0.036\%, 0.057\%, 0.109\%, 0.192\%, 0.340\%, 0.803\%\}$. *Source: Campbell et al.* [24].

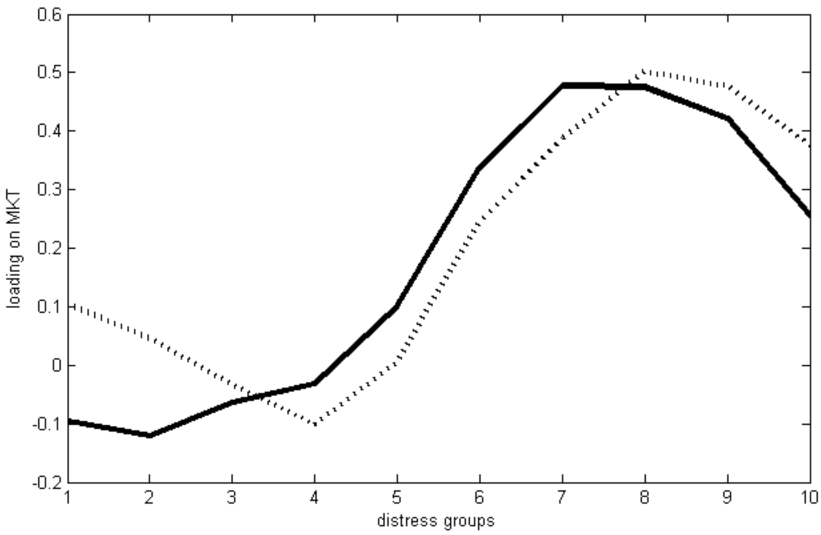

Figure 6. The loadings on the market for the ten portfolios of firms sorted with respect to the *per year* probability of default. The probabilities of default are as in Figure 5. The solid line is the loading on MKT in a three factor model of Fama and French [33], and the dotted line is the loading in the CAPM (one factor) model (computed from realized returns and alphas in figure 5, assuming mean market excess return of 0.0575.). *Source: Campbell et al.* [24].

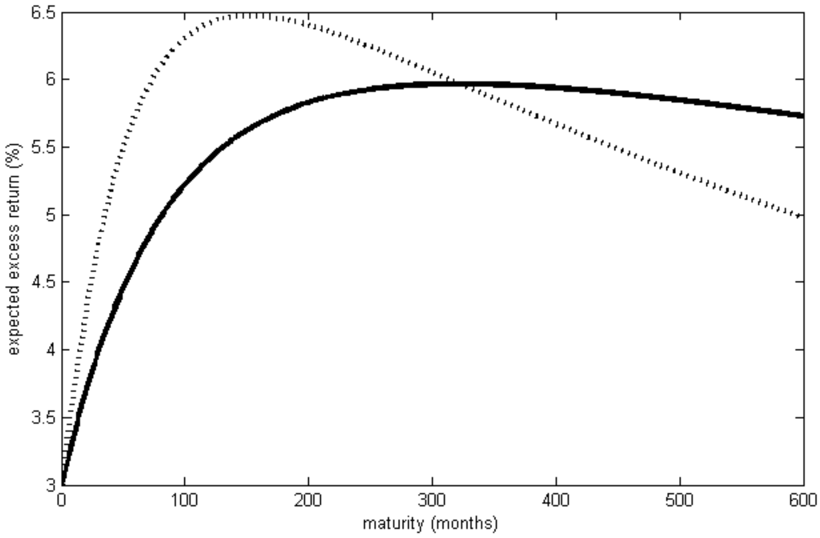

Figure 7. Expected excess returns on *cumulative equity strips* (solid line), defined as cumulative means of expected excess returns of the individual strips (dotted line).

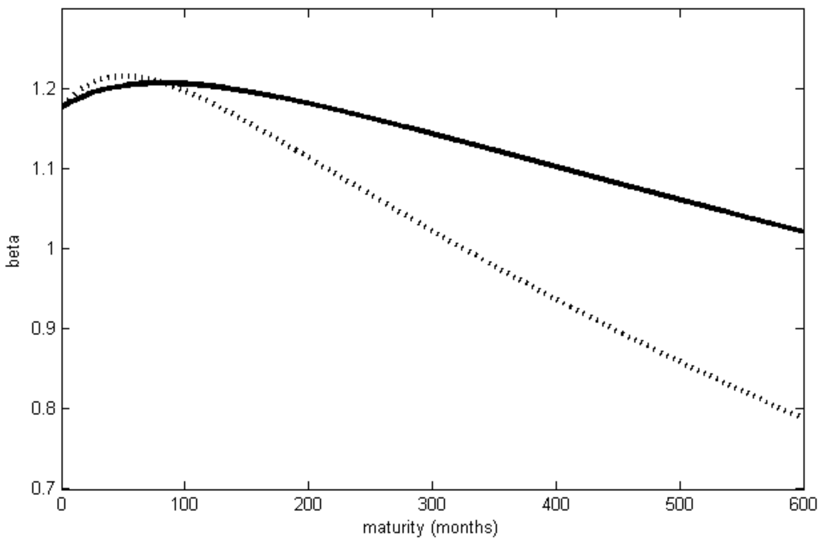

Figure 8. Market betas of *cumulative equity strips* (solid line), defined as above, compared with the betas of individual strips (dotted line).

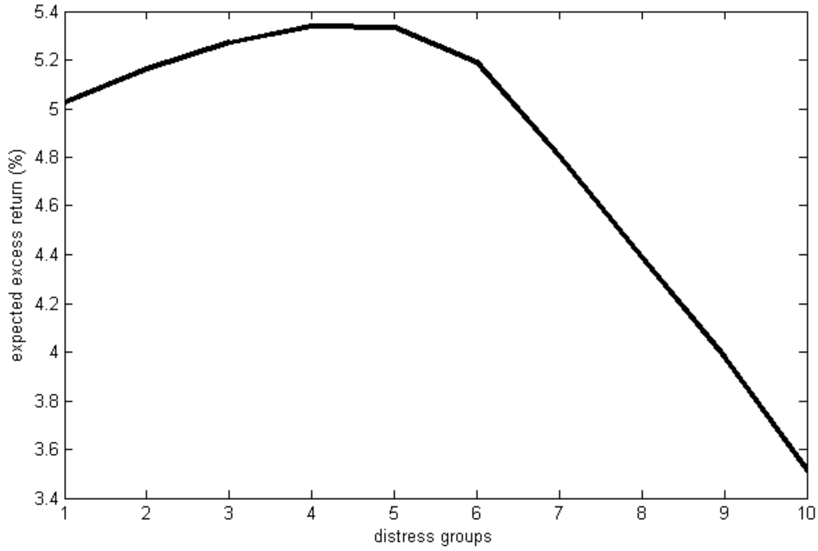

Figure 9. Model-implied unconditional expected excess returns of ten firms with *per year* probabilities of default as in figure 5. The most distressed firms are on the right.

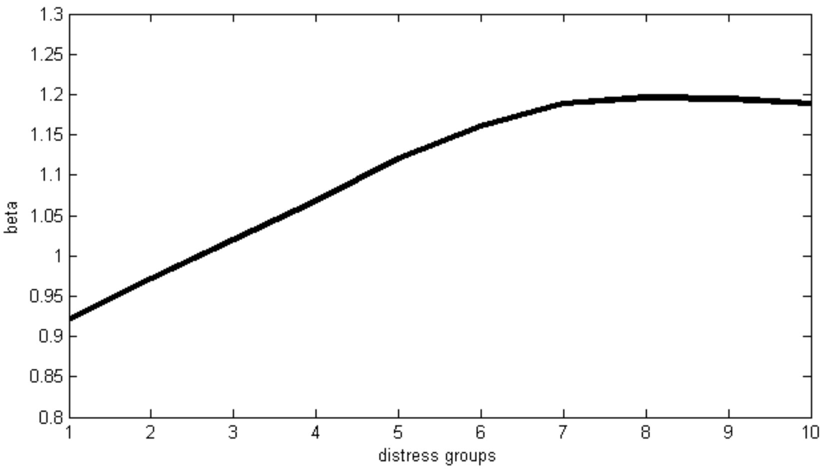

Figure 10. Model-implied unconditional market betas of ten firms with *per year* probabilities of default as in figure 5. The most distressed firms are on the right.

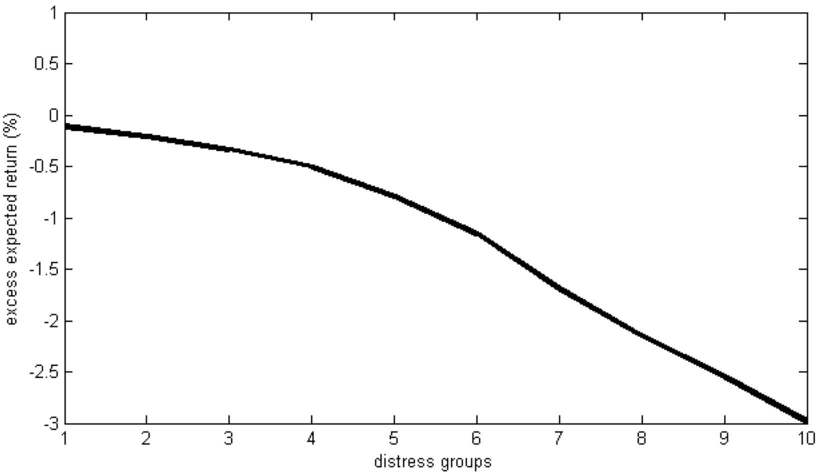

Figure 11. Model-implied CAPM alphas of ten firms with *per year* probabilities of default as in figure 5. The most distressed firms are on the right.