AN ANALYSIS OF ZERO-COUPON FORWARD PREMIA

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Abstract. I apply a simple, model-independent procedure to extract forward premia from the term structure of zero coupon bonds, using only the assumption of rational expectations of bond market participants. Principal component analysis uncovers the factor structure in the extracted premia. 89% of their unconditional variance is explained by a single factor, which is related to the tent-shaped linear combination of forward rates familiar from Cochrane and Piazzesi ([5]). The second principal component accounts for over 5% of the variation and both of them seem to be related to the business cycle. I try to relate the first two principal components to macroeconomic variables and find that the first can be explained by stock return forecasting variables (price-earnings ratio and *cay*) but also dollar index and a measure of sentiment from Baker and Wurgler ([2], [3]). The second seems to be more directly related to real activity, as measured by PMI index and total capacity utilization. The dominant factor helps explain the cross sections of equity sorted by value, size, price-earnings and momentum. I do not find evidence that value premium is related to the bond market, instead I find such a relation for the size premium.

1. Introduction

The central focus of asset pricing is to identify economic risks and corresponding risk prices that together determine risk premia, or equivalently expected returns on assets. In this paper I make a step towards understanding the premia on the bond market.

What makes this market special is the presence of traded claims to cash flows of different maturities. The prices of these *zero-coupon bonds* define the term structure of interest rates. A forward rate is defined as the rate at which two parties commit to borrow and lend money at future date, for given time. Forward rates for short term loans can be decomposed into expected future short rates plus term premia that represent the risks that the parties take into account at the moment of the agreement. If this decomposition was observable, we would know what risk premia are associated with different cash flow maturities. Without this knowledge, one can only try to infer the premia from observed data using a set of assumptions.

One approach is to construct and calibrate a model with explicitly specified risk factors and prices of risk. If the model is specified correctly, the prices of risk can be inferred with very high accuracy. However, in practice both the risk factors and the premia are unknown, which has led researchers to assume some structure first. This problem is well illustrated by Duffee ([9]), who notes that for a five-factor affine model there are as many as 52 free parameters. Without assuming some structure, efficient estimation becomes very difficult, if not impossible.[1]

In this paper I apply a model-independent procedure to extract forward premia from the term structure of forward rates, using only the assumption on rational expectations of bond market participants. I use the (unsmoothed) Fama-Bliss ([11]) dataset with period coverage identical to Cochrane-Piazzesi ([5]). These data inform about the prices of zero

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[1]Principal component analysis of the yield or forward curve indicates that no more than 3 factors are enough to capture the cross section (with respect to maturity) of the term structure of interest rates *at every point in time.* However, Cochrane and Piazzesi ([5]) show that factor four and five, although virtually absent from the point of view of the cross section, raise significantly predictive R-squared in expected bond return regressions. This has led Duffee ([9]) and Joslin, Priebsch and Singleton ([12]) to include factors that are orthogonal to the term structure at a given point of time but affect the direction of its future movements (and equivalently excess bond returns). Duffee assumes that this missing factors are latent, while Joslin, Priebsch and Singleton build a model in which the information about them is carried by macroeconomic variables.

coupon bonds with maturity one through five years, at the end of every month. I analyze five one-year forward rates (that can be computed from the prices), corresponding to one-year loan contracts valid zero through four years ahead.

I define the n-th forward premium slightly differently than most of the literature does – as the difference between current n-year forward rate and expected n-1-year forward rate one year ahead. The forward premium defined this way can be interpreted as the amount of priced risk that affects n-th period bond and that is expected to be resolved during the coming year. The cross section of such forward premia has immediate implication for one-year expected excess holding returns of all maturity bonds.

I perform a principal component decomposition of the forward premia and find that the first factor accounts for 89% of their unconditional variation and the second for 5%. The properties of the first component allow for its loose interpretation as the single factor from Cochrane and Piazzesi ([5], [6]) – both affect expected bond returns of all maturities in a similar way and both are explained well (and with similar R^2) by a tent-shaped linear combination of forward rates. Interestingly, the shape of this combination is robust to the inclusion of macroeconomic variables that explain the analyzed component of forward premia beyond forward rates. This findings suggest that the bulk of variation in expected bond returns is indeed driven by a single factor, as suggested by Cochrane and Piazzesi ([5]). However, the other three principal components of the risk premia sum up to a significant 11% of their total variation, which makes them difficult to ignore. It turns out that the second principal component of forward premia is only weakly related to forward rates (with coefficients not forming a tent anymore) and that macroeconomic variables related to economic activity help explain a significant part of it.

The relation between bond excess returns (and therefore forward premia) and macroeconomic variables has been recently explored in several studies. Ang and Piazzesi ([1]) include them directly into the set of state variables so that they interact with latent yield curve factors. Ludvigson and Ng ([18]) perform a principal component analysis of a very large number of macroeconomic time series and find that five or six components significantly help in predicting expected bond returns.[2]

In this paper, I also try to clarify the links between the cross sections of stocks and bonds that have been recently studied by Koijen et al. ([13]) and Baker and Wurgler ([3]). Koijen et al. find that value stocks (with high book-to-market ratios) have higher exposure to shocks to the Cochrane and Piazzesi (CP) factor than growth stocks, and this slope in the exposures is the key for understanding the value premium in their model. On the other hand, Baker and Wurgler analyze the exposure of book-to-market deciles with respect to realized returns of long maturity bonds. They find that the exposures are negative (all deciles covary negatively with long bonds), but the magnitude is the highest for extreme deciles on both sides. Thus, a hump-shaped relation is found, which leads the authors to the conclusion that the properties of the cross section of value-sorted stocks is hard to explain using bond market alone. Instead, they focus on a different dimension of the data, arguing that there are "bond-like" and "speculative stocks". The former are characterized by the property that they lose value precisely when bonds gain value, consistently with the concept of a "flight to quality". They find that stocks that are "bond-like" are those that are bigger (in terms of market capitalization), less volatile, dividend-paying, older and not distressed. In this study, I find no evidence that bond market factors explain high returns of value stocks versus growth stocks. Instead, the results suggest that such link exists for the cross section of stocks sorted by size and earnings-to-price ratio. Importantly, only the first component of forward premia is found to be significant in explaining these cross-sections. Another, more puzzling finding, is that this component is found to predict stock portfolios formed by momentum and that it predicts both extreme winners and losers in the same direction, having no significant impact on intermediate portfolios.

On the theoretical side, the links between stocks and bonds have been investigated by Lettau and Wachter ([17]). Their modeling assumption is that stocks with high book-to-market are the same as stocks with high earnings-price ratio. The findings in this

[2]Ludvigson and Ng ([18]) assume that a single factor drives bond returns. The six principal components are then used to explain its variation.

paper and in Baker and Wurgler ([3]) seem to suggest that this can only be a first-order approximation.

The next section presents the methodology that is used to extract the forward premia. Section 3 presents the results of principal component decomposition and describes the basic properties of the components. Section 4 tries to identify macroeconomic variables that are helpful in statistical explanation of forward premia. In section 5, I use the principal components of extracted forward premia in regressions relating them to the excess returns of stock portfolios sorted with respect to various characteristics.

2. Extracting Forward Premia

The problem of extracting forward premia from the term structure of interest rates is central to understanding how risk is priced in the bond markets. Usually, researchers rely on models that, after estimation, allow to decompose the term structure of interest rates into expected future short rates and term premia.[3]

This study takes a different approach. I use the definition of bond excess returns to extract forward premia using the assumptions of rational expectations of bond market participants. Expected excess returns from holding bonds of various maturities from time t to $t+12$ (one year) are unobservable to the econometrician. Hovewer, the *realized* returns are known at $t+12$, and they can be used by him as the unbiased estimates of expected returns perceived by the agents (equivalently, one can assume that the agents form rational predictions about next year's forward rates).

I use the Fama-Bliss ([11]) monthly data on bond prices with maturities 1 through 5 years, for the time span July 1952 – December 2003.[4] Let p_t^n denote log price of zero-coupon bond at time t, maturing n-years later. The forward rates (for one-year loans entered n-1 years from today) are defined as

$$f_t^n \equiv p_t^{n-1} - p_t^n, \tag{1}$$

one year holding returns of n-maturity bond are

$$r_{t+1}^n \equiv p_{t+1}^{n-1} - p_t^n,$$

and excess returns over 1-year risk free rate are[5]

$$rx_{t+1}^n \equiv p_{t+1}^{n-1} - p_t^n - f_t^1. \tag{2}$$

Equation (1) tells us that n-th forward rate is the negative of marginal (one-year) increment of the discount function at maturity n. In other words, log bond prices are cumulative sums of forward rates (with negative sign). Stacking forwards and log prices in (horizontal) vectors, one can write equation (1) as

$$[f_t^1, f_t^2, f_t^3, f_t^4, f_t^5] = [p_t^1, p_t^2, p_t^3, p_t^4, p_t^5] \times \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}.$$

Inverting the relation above gives

$$[p_t^1, p_t^2, p_t^3, p_t^4, p_t^5] = -[f_t^1, f_t^2, f_t^3, f_t^4, f_t^5] \times \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

Substituting bond prices with indices n-1 and n from (3) into equation (2) yields

$$rx_{t+1}^n = \sum_{i=2}^{n} f_t^i - \sum_{i=2}^{n} f_{t+1}^{i-1}. \tag{4}$$

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[3]Examples of affine term structure models are Duffie and Kan ([10]), Dai and Singleton ([7]), Duffee ([8]) and Piazzesi ([19]).

[4]The data are available on John Cochrane's webpage.

[5]I use the return on one-year bond as the risk-free rate, following Cochrane and Piazzesi ([5]),([6]).

Excess one-year bond returns depend, by definition, on the current set of forward rates, and the corresponding set of forward rates next year, with the index decreased by one.

I define the n-th forward premium as the difference between the n-th forward rate today and the agents' time-t expectation of (n-1)-th forward rate next year:

$$g_t^n = f_t^n - E_t^a[f_{t+1}^{n-1}]. \tag{5}$$

This definition[6] allows to rewrite (4) as

$$rx_{t+1}^n = \sum_{i=2}^{n} g_t^i - \sum_{i=2}^{n}\left(f_{t+1}^{i-1} - E_t^a[f_{t+1}^{i-1}]\right). \tag{6}$$

In a risk-neutral world, the expectations hypothesis holds, and forward rates are the unbiased predictors of future short rates. Equivalently, they are unbiased predictors of next year's forward rates with maturity n-1, so the forward premia in (5) and expected excess bond returns that follow from (6) would be zero. In a world with risk aversion but constant risk premia, future short rates would still be predictable by todays forward rates (after subtracting the premium). In the real world however, forward premia are not only positive, but time-varying. This is equivalent to the statement that the expectations hypothesis fails, which is strongly supported by the data.[7]

Equation (6) shows that there are two reasons for the variation in excess bond returns. One is that the forward premia are changing, and the other is that next year's realized forward rates differ from expectations. The former is the source of variation in *expected* bond returns, while the latter accounts for the *unexpected* part.

Neither forward premia, nor agent's expectations about future forward rates are known to the econometrician. However, if agents have rational expectations, then realized economic variables should on average be equal to their expectations. Define $E_{t+1}^e[Z_t]$ to be the time t+1 (ex post) econometrician's expectation about random variable Z_t that is known to agents at time t. Applying this operator to equation (6) and using the rational expectations assumption yields

$$rx_{t+1}^n = E_{t+1}^e[rx_{t+1}^n] = \sum_{i=2}^{n} E_{t+1}^e[g_t^i] - \sum_{i=2}^{n} E_{t+1}^e\left(f_{t+1}^{i-1} - E_t^a[f_{t+1}^{i-1}]\right) = \sum_{i=2}^{n} E_{t+1}^e[g_t^i].$$

Defining $\hat{g}_t^i \equiv E_{t+1}^e[g_t^i]$, and stacking estimated forward premia and excess returns together, above expression means the same as

$$[rx_t^2, rx_t^3, rx_t^4, rx_t^5] = [\hat{g}_t^2, \hat{g}_t^3, \hat{g}_t^4, \hat{g}_t^5] \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{7}$$

which can be inverted to obtain

$$[\hat{g}_t^2, \hat{g}_t^3, \hat{g}_t^4, \hat{g}_t^5] = [rx_t^2, rx_t^3, rx_t^4, rx_t^5] \times \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{8}$$

I will refer to the set of $\hat{g}_t^n$'s as the "implied forward premia", or simply "forward premia".[8] One can see that their extraction using the rational-expectations assumption is similar to taking realized bond excess returns as proxies for their expectations. My work goes just a little bit further and takes cross-sectional (across bond maturities) differences in these realized returns as the main focus of interest. This can be justified if one realizes that forward premium with index 2 influences expected return of *all* bonds, the premium with index 3 affects all but the one with index 2; and so on. Forward premia are thus more "disaggregated" than excess returns, which makes them a better object of analysis.

[6] The term "forward premium" is sometimes used in a different context in the literature – as the difference between the n-th period forward rate and expected short rate *n years later*.

[7] See Fama and Bliss ([11]), Campbell and Shiller ([4]), and many others.

[8] The premium with index 1 is missing in (7), because it is associated with f_t^1, which is the risk-free rate.

This way of extracting forward premia has a great advantage of being model-free. There is also a disadvantage, as using realized forward premia in place of their expectations necessarily introduces an error, which is identical to the forecasting error made by the market participants at the moment of the formation of expectations. On top of that, if this forecast error from month t to $t+12$ is positive, it is also likely to be positive between $t+1$ to $t+13$, so the errors are autocorrelated.

To defend my approach, I note that using realized forward premia does not in fact differ from the generally accepted practice of using realized returns in place of expected returns by a vast majority of empirical studies that attempt to explain time-series predictability of asset returns. It can also be argued that results obtained with noisy data can still be interesting, provided that the noise itself is not driving them. In my case, the noise is the difference between realized and predicted values, which should be (under my assumption of rational expectations) orthogonal to all information available at the time of prediction. This allows me to trust regressions that explain forward premia from time t to $t+12$ by variables known at t. Moreover, my approach makes use of rational expectations in *one-year-ahead* predictions of forward rates by current forward rates, as opposed to *n-year-ahead* predictions of short rates by current forward rates. With this relatively short horizon, the assumption is more likely to hold.

3. Principal Component Analysis

I decompose the unconditional covariance matrix of forward premia obtained in (8) to get an insight of what is the factor structure that drives their time-variation. The eigenvectors associated with the factors are in Figure 1. It is easy to re-interpret the eigenvectors seen

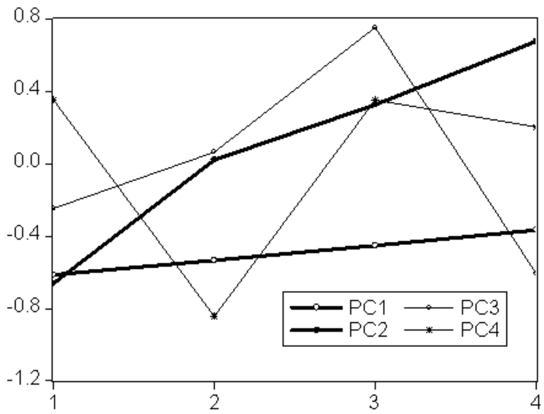

Figure 1. Eigenvectors in principal component analysis of forward premia. PC1 is the eigenvector associated with the largest eigenvalue.

in the picture in terms of expected bond returns: using relation (7) – the vector of expected bond returns corresponding to a given eigenvector of forward premia is just the cumulative sum of the components of this eigenvector. For example, the first eigenvector of the premia is $[-0.61, -0.53, -0.54, -0.37]$, which corresponds to a vector $[-0.61, -1.14, -1.68, -2.05]$ of expected one-year bond returns for maturities 2-5 years. When the first factor behind forward premia moves by a unit, expected returns of all maturity bonds go down. Analogously, the vector of expected bond returns associated with the other highlighted eigenvector is $[-0.66, -0.64, -0.31, +0.37]$ – when the second most important factor driving forward premia changes by unit, it implies that expected returns of short-maturity bonds go down, while those of bonds with maturity above 4 years go up.

The first two principal components (with eigenvectors highlighted in the graph) account for respectively 89.8% and 5.2% of the total variance of the forward premia and the other two add 2.9% and 2.0%. This is in contrast to finding of Cochrane and Piazzesi ([5]) who find that the variation in expected returns of all bonds is explained in 99.5% by a single factor. They are working with bond returns directly, so in light of the previous paragraph their finding is not surprising: since the first PC of forward premia drives returns of all bonds in the same direction, the variation induced by other factors may not be captured well by the PC decomposition done on bond returns.

The finding of more than one independent factor behind expected bond returns is supported by other studies. For example, Joslin, Priebsch and Singleton ([12]) find this dimension to be at least 2. In the present context, the question is whether the remaining components of forward premia (that jointly account for 10.2% of their total variation) can be considered important, and if yes, what are economic forces that drive them.

4. Forward Premia and Economic Variables

I start the analysis of the forward premia factor scores by plotting them against the NBER recession indicator. In the next subsection, I try to link the premia to macroeconomic variables.

4.1. **Graphical Interpretation.** Figure 2. suggests that the first score seems to be related to the business cycle, being lower in recessions. This pattern is clearer before around 1983. The correlation coefficient with the NBER recession index is -0.17. Figure

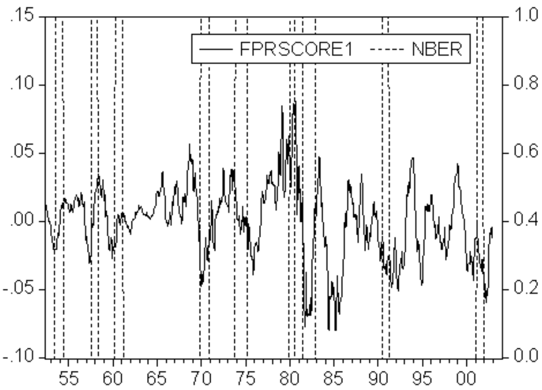

Figure 2. Factor score of the first most important principal component of forward premia, plotted against the NBER recession indicator.

3. shows that the factor score corresponding to the second principal component is also business-cycle related. This relationship is more difficult to notice from the picture alone because of higher conditional volatility of the series. However, the correlation with the indicator is even slightly higher (-0.22) than in the previous case, which is due to the fact that the relation with the business cycle seems to exist for the whole sample. Factor scores

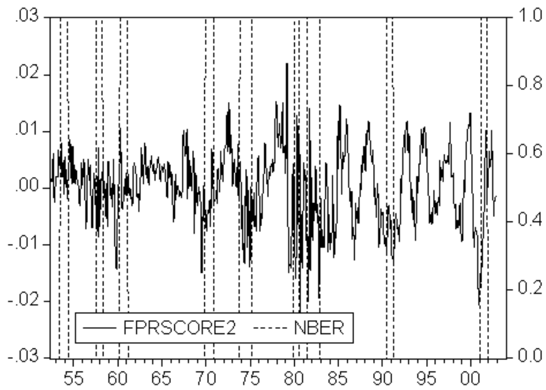

Figure 3. Factor score of the second most important principal component of forward premia, plotted against the NBER recession indicator.

with numbers 3 and 4 seem to be almost uncorrelated with the business cycle (correlation coefficients -0.10 and 0.02, respectively).

In the analysis that follows I concentrate on the first two factors of forward premia. I do so for several reasons. Firstly, the inspection of eigenvectors in the previous subsection allows for an intuitive interpretation of risk premia of bonds with various maturities that correspond to the first two eigenvectors. Secondly, the same analysis of eigenvectors and correlation with the business cycle of factor scores associated with factors 3 and 4 cast doubt on whether they are interpretable and economically important or whether they represent noise. Finally, the first two principal components account for as much as 95% of total variation in implied forward premia.

4.2. **Relation to Macroeconomic Variables.** I use monthly data on several economic variables that have been found in the literature, or can be believed to predict stock or bond excess returns – equivalently to carry information about risk premia. Stock market aggregate price-earnings ratio *pe* is from Robert Shiller's web page. *caym* is a monthly version of *cay* from Lettau and Ludvigson ([14], [15]).[9] The variables *dollar*, *napm*, *fedfund* and *tcu* are respectively: the dollar index, measuring the strength of the dollar as compared to a basket of foreign currencies, PMI manufacturing index published by the Institute of Supply Management (ISM), effective fed funds rate and total capacity utilization in the industry. All of them can be downloaded from St. Louis Fed database (FRED). *sent* is the sentiment index constructed and used by Baker and Wurgler ([2], [3]) and available on Jeffrey Wurgler's web page. Finally, *infl* and *ipgr* are 12-month growth rates of consumer price and industrial production indices, that are available on FRED.[10]

I start the analysis by identification of variables that influence the first two principal components of the forward premia. The first component is significantly explained by variables appearing in Table 1, while the second by those in Table 2. In both cases I started with a regression that used all of the variables on the right hand side and eliminated insignificant ones one at a time, starting from those with the highest p-values.[11]. Because of the unavailability of the *tcu* series prior to 1967, the time series used are for the years 1967-2002.[12]

	f1	f2	f3	f4	f5	pe	caym	dollar	sent	infl	ipgr	$\bar{R}^2$
a)	1.63	-0.61	-2.18	-0.63	1.48							0.33
	(0.00)	(0.16)	(0.00)	(0.02)	(0.00)							
b)	1.42	-0.19	-2.08	-0.87	01.04	-1.3e-3						0.41
	(0.00)	(0.63)	(0.00)	(0.02)	(0.00)	(0.00)						
c)	1.53	-0.56	-2.27	-0.50	1.51		-0.21					0.34
	(0.00)	(0.18)	(0.00)	(0.05)	(0.00)		(0.02)					
d)	1.56	-0.65	-1.86	-0.69	1.36			-4.6e-4				0.36
	(0.00)	(0.12)	(0.00)	(0.01)	(0.00)			(0.00)				
e)	1.60	-0.64	-2.08	-0.69	1.46				.63e-3			0.34
	(0.00)	(0.13)	(0.00)	(0.01)	(0.00)				(0.01)			
f)	1.08	-0.72	-1.46	-0.66	1.28					0.39	0.14	0.39
	(0.00)	(0.08)	(0.00)	(0.01)	(0.00)					(0.00)	(0.00)	
g)	0.83	-0.16	-1.60	-0.71	0.82	-1.5e-3	-0.81	7.9e-4	9.8e-3			0.52
	(0.00)	(0.66)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)			
h)	0.85	-0.30	-1.42	-0.76	0.84	-1.6e-3	-0.78	7.6e-4	9.7e-3	4.3e-3	0.07	0.53
	(0.00)	(0.40)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.95)	(0.02)	

Table 1. Coefficients from the regression of the form $fprpc1 = const + c_1 f_1 + c_2 f_2 + c_3 f_3 + c_4 f_4 + c_5 f_5 + c_6 pe + c_7 \text{caym} + c_8 dollar + c_9 sent + c_{10} infl + c_{11} ipgr$, where $fprpc1$ is the first principal component of the forward premia (the one which accounts for the most of their variation). I report p-values and adjusted R^2. I do not report the constant.

In Table 1, I report an analysis of the first principal component of the forward premia. Row a) reports regression coefficients when only forward rates are used. The adjusted R^2 is 0.33, which is similar to 0.31 – 0.37 for excess bond returns for Cochrane and Piazzesi ([5]) paper.[13] The notable feature of the regression is that the coefficients on the forward

[9]The original data on *cay* are quarterly. *cay* id definad by Lettau and Ludvigson as the deviation from long-run cointegrating relation between consumption and wealth, which is measured as weighted average of asset- and human wealth. I compute a monthly version of *cay* by linear interpolation.

[10]Using monthly rates of inflation and industrial production results in less significant estimates of the corresponding coefficients.

[11]As a robustness check I tried different orders of elimination, with the same results.

[12]This is slightly shorter than 1964-2003 in Cochrane and Piazzesi ([5]).

[13]These R^2's are not directly comparable because of different left-hand side variables. However, since the first principal component of forward premia explains the bulk of their unconditional variation, it is also the main driving force of excess returns for all-maturity bonds.

rates form the "tent", also familiar from their paper, which is inversed here because of the negative sign of the loadings on the factor score. Interestingly, the tent-like pattern does not disappear after controlling for other variables in rows b) through h), which suggests that it is not an artifact of an omitted variable bias. These two findings suggest that Cochrane and Piazzesi ([5]), by restricting all bond returns to be driven by a single factor, effectively analyze the first principal component of the forward premia.[14]

The rows b) through f) of table 1 analyze explanatory power of additional macro variables. For the sample period being analyzed, the equity price-earnings ratio increases predictive R^2 by 8%, while the other three variables by $1-3\%$ each. All four raise the R^2 up to a remarkable 52%. Past inflation and industrial production growth are used by Joslin et al. ([12])[15] to show that with these variables, principal components 4 and 5 of the forward curve, found by Cochrane and Piazzesi ([5]) to add as much as 9 percentage points to their predictive regression, become insignificant.[16] I find that these variables help explain the most important principal component of the forward premia by adding 6% to the regression R^2, but only when the other variables are not included. In the latter case, the impact is very weak.

Let us now take a look at the signs of coefficients. The price-earnings ratio enters with negative sign, which means that its growth leads to a rise in expected bond returns, as measured by the first PC of the forward premia. This variable is known to be a predictor of low expected stock returns at long horizons, although slightly weaker than the p/d ratio. At one-year horizeon, p/e forecasts excess equity return with an R^2 of around 3%. Thus, the results here suggest that price-to-earnings predicts the returns of debt and equity to move in opposite directions, which suggests that the variation in this variable can reflect low-frequency fluctuations in market risk aversion, which makes investors shift from debt to equity and vice-versa. The situation is different with *caym*, which is known to forecast equity returns at one-year horizon with an R^2 much higher than that of p/e or p/d ratios.[17] In my sample, *caym* forecasts expected returns on bonds and stocks in *the same* direction, which is consistent with its interpretation as a variable that moves at higher, business-cycle frequency. However, its incremental explanatory power for bond returns is small (it increases the R^2 by roughly 1%).

The relation between the first PC of forward premia and the dollar index suggests that expected bond returns are low when the dollar is relatively strong compared to other currencies, but the coefficient seems to be unstable across the specifications and it changes the sign. Finally, the measure of investor sentiment seems to be negatively related to expected bond returns related to the analyzed PC.[18]

The analysis in Table 2. is analogous, with the exception that the object of analysis is the second principal component of the forward premia, and different variables are found to be significant on the right hand side. Here, the R^2's are lower and coefficients on forward rates estimated with much less precision. The "tent" does not show up, but the data suggest a monotone pattern of coefficients instead: negative values can be seen at shorter forwards and positive at longer ones. Notably, forward rates perform very poorly in explaining the analyzed factor. The R^2 is just 11%, which may be another explanation of why Cochrane and Piazzesi ([5]) found only one factor using forward rates only. However, macroeconomic variables add significant explanatory power to the regression. With the exception of the effective fed fund rate, these variables are strongly related to business-cycle frequency fluctuations in output. As in the previous case, inflation and industrial

[14]They admit that this one-factor restriction is statistically rejected. My work suggests that the reason of the rejection may be precisely the statistical significance of other factors in the forward premia.

[15]Their data span is January 1989 – June 2008. They construct the zero-coupon yield curve using swap market quotes.

[16]In an unreported regression on unsmoothed Fama-Bliss data for time period much longer than in Joslin at al., I found that only factor 5 becomes insignificant. This suggests that there must be factors spanned neither by inflation nor forward rates that influence the time series of bond returns.

[17]This R^2 is about 18%. See Lettau Ludvigson ([15]).

[18]The measure of sentiment used here is the one that is constructed without orthogonalization to the macro variables. Baker and Wurgler provide alternative measure based on series orthogonalized with respect to inflation, IP growth and other macroeconomic series. Using the latter barely changes the results.

	f1	f2	f3	f4	f5	napm	fedfund	tcu	infl	ipgr	$\bar{R}^2$
a)	-0.06	-0.03	-0.26	0.09	0.23						0.08
	(0.26)	(0.80)	(0.01)	(0.20)	(0.00)						
b)	-0.07	-0.10	-0.08	0.06	0.18	3.8e-4					0.20
	(0.20)	(0.35)	(0.40)	(0.38)	(0.00)	(0.00)					
c)	0.38	-0.22	-0.34	0.16	0.26		-2.4e-3				0.16
	(0.00)	(0.06)	(0.00)	(0.02)	(0.00)		(0.00)				
d)	-0.24	-0.08	-0.05	0.09	0.32			1.0e-3			0.23
	(0.00)	(0.45)	(0.61)	(0.19)	(0.00)			(0.00)			
e)	-0.08	-0.15	-0.08	0.06	0.24				0.02	0.06	0.17
	(0.17)	(0.18)	(0.39)	(0.36)	(0.00)				(0.19)	(0.00)	
f)	0.11	-0.23	-0.09	0.12	0.30	1.4e-4	-1.7e-3	7.7e-4			0.29
	(0.21)	(0.03)	(0.33)	(0.06)	(0.00)	(0.01)	(0.00)	(0.00)			
g)	0.09	-0.22	-0.07	0.13	0.29	1.8e-4	-1.7e-3	8.1e-4	0.01	-8.6e-3	0.29
	(0.33)	(0.05)	(0.43)	(0.05)	(0.00)	(0.01)	(0.00)	(0.00)	(0.41)	(0.51)	

Table 2. Coefficients from the regression of the form $fprpc2 = const + c_1 f_1 + c_2 f_2 + c_3 f_3 + c_4 f_4 + c_5 f_5 + c_6 napm + c_7 fedfund + c_8 tcu + c_9 infl + c_{10} ipgr$, where $fprpc2$ is the second principal component of the forward premia. I report p-values and adjusted R^2. I do not report the constant.

production contribute significantly to the regression, but they are driven out by the set of the three other variables, which can be seen in rows e) and g).

Concerning the signs of the coefficients, remember that when the second PC of the forward premia increases, expected returns of 2- and 3-year bonds go down, that of the 4-year bond remains unchanged, and those on longer bonds go up. As seen from the risk-based perspective, it could mean that investors become less concerned about the long term risk relative to the short term. This can happen when the economy is doing well, which explains positive signs on the manufacturing PMI index and the total capacity utilization. The sign of the effective fed fund rate is more puzzling, especially if we assume that this rate should be pro-cyclical. However, in the sample that is being analyzed, it is very likely that the *nominal* rate is counter-cyclical, due to high inflation in the 70's and 80's. Concerning the real interest rate, some papers argue that it can also be modeled as counter-cyclical.[19]

5. Relation to the Cross Section of Equity

The next step in the analysis is to test whether extracted components of forward premia have an explanatory power for the cross section of equity. I use four sorts of equity used previously in the literature: with respect to the book-to-market equity, earnings-to-price ratio, size and momentum. The data on the portfolios are from Kenneth French's website.

In unreported set of regressions I verify that principal components other than the first (most important) are statistically insignificant when applied to explain all cross-sections. Therefore, I run regressions of the form

$$\text{realized return within the decile} = \alpha + \beta \text{factor score of the first principal component},$$

using only the first principal component on the right.

The results for all of the sorts are in tables 3 – 6. The betas are positive, indicating that expected returns on all portfolios are larger when the first principal component of forward premia is larger. Remembering that the component is high when expected bond returns are low, this leads to a conclusion that all of the portfolios have negative coefficients on expected bond returns implied by this component, which is a confirmation of the results of Baker and Wurgler ([3]). Accordingly, portfolios with higher beta are more "speculative".[20]

The results for the value sort are in table 3. Consistently with the findings of Baker and Wurgler ([3]), extreme deciles on both sides are more "speculative". The difference between their study and mine is that instead of long-term bond return, I use a right-hand-side variable that is effectively the component of return that is common to all-maturity bonds.

[19] See Wachter ([20]). In her habit-based model, the real rate is countercyclical, because interest rate responds positively to a decrease in the surplus consumption ratio. This decrease is considered as a "bad" time by the representative agent, who wants to borrow from the future to smooth the path of utility. This effect dominates a decrease in the interest rate, which results from increased precautionary savings.

[20] Or, according to a risk-based interpretation of this result – they are more risky and their price drops the most when the bad times come.

	1	2	3	4	5	6	7	8	9	10
α	0.89	0.96	1.00	0.97	1.09	1.11	1.10	1.25	1.26	1.32
β	20.1	15.2	14.3	14.5	14.1	16.6	13.9	11.4	14.6	19.5
	(0.01)	(0.02)	(0.03)	(0.03)	(0.02)	(0.00)	(0.02)	(0.05)	(0.02)	(0.01)

Table 3. Alphas and betas from the regression $return_i = \alpha + \beta fprpc1$, where $return_i$ is the monthly excess portfolio return within i-th decile of the book-to-market sort and $fprpc1$ is the dominant principal component of forward premia. p-values in parentheses.

The sort with respect to earnings to price reveals a clearer pattern, despite the fact that for three highest deciles the estimates are imprecise. The betas are higher for the portfolios that contain stocks with low earnings-to-price ratios. Some authors suggest that sorting with respect to earnings-to-price is very close to sorting by market-to-book,[21] while tables 3 and 4 show that these two cross sections differ in an important way – by how their returns correlate with the bond market.

	1	2	3	4	5	6	7	8	9	10
α	0.87	0.82	0.98	0.98	1.02	1.21	1.24	1.34	1.42	1.43
β	27.6	17.5	14.0	14.0	14.7	13.3	13.2	9.9	11.3	14.5
	(0.00)	(0.01)	(0.03)	(0.02)	(0.02)	(0.03)	(0.03)	(0.11)	(0.08)	(0.04)

Table 4. Alphas and betas from the regression $return_i = \alpha + \beta fprpc1$, where $return_i$ is the monthly excess portfolio return within i-th decile of the earnings-to-price sort and $fprpc1$ is the dominant principal component of forward premia. p-values in parentheses.

The sort with respect to size dimension, reported in table 5, again confirms Baker and Wurgler ([3]). Small stocks have high expected return over the next month precisely when bonds have low expected returns. As before, there can be two alternative explanations. According to the behavioral one – small stocks are sold by investors in the course of flight to quality. The risk-based explanation would state that such a situation can happen when market risk aversion increases and if small stocks are the most risky ones, so that their price drops the most, while the price of assets that provide hedge (bonds) goes up.

	1	2	3	4	5	6	7	8	9	10
α	1.20	1.19	1.20	1.15	1.17	1.10	1.12	1.08	1.02	0.94
β	29.1	24.5	25.5	24.1	24.6	20.1	20.0	17.9	15.1	13.9
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.02)	(0.01)

Table 5. Alphas and betas from the regression $return_i = \alpha + \beta fprpc1$, where $return_i$ is the monthly excess portfolio return within i-th decile of the size sort and $fprpc1$ is the dominant principal component of forward premia. p-values in parentheses.

The last table presents the result for the portfolios of stocks sorted with respect to the past returns. It is interesting that both extreme winners and losers covary with bonds in the similar way, while the middle deciles have insignificant betas.

	1	2	3	4	5	6	7	8	9	10
α	0.09	0.62	0.79	0.85	0.86	0.96	1.00	1.18	1.27	1.17
β	39.6	19.5	14.0	9.4	10.0	12.2	11.4	16.7	18.5	27.6
	(0.00)	(0.02)	(0.04)	(0.13)	(0.10)	(0.05)	(0.06)	(0.01)	(0.01)	(0.00)

Table 6. Alphas and betas from the regression $return_i = \alpha + \beta fprpc1$, where $return_i$ is the monthly excess portfolio return within i-th decile of the momentum sort and $fprpc1$ is the dominant principal component of forward premia. p-values in parentheses.

[21]See, for example, Lettau and Wachter ([16]), who argue that value stocks are those that have their dividend payments shifted towards the present, while growth – more to the future. In their paper, value and earnings-to-price dimensions are the same.

Overall, the evidence presented in this section suggests that there may be a non-monotonic relation between the most important principal component of the premia in the bond market and expected returns of stock sorted by the value characteristics. The dimension that seems to be quite strongly related to bond premia is size. Large stocks do covary negatively with bonds, but not as much as small stocks. It is a question for future research whether small stocks are more risky, or whether they are the most mispriced ones during swings in investors' sentiment. It is also puzzling that both winner and loser stocks perform poorly when bonds perform well. Explanation of this fact is left for future research.

6. Conclusions

The contribution of this paper is threefold. First, I propose a model-free way to extract one-year bond market forward premia using the data on realized bond returns and assumption on rational expectations of market participants. This approach is independent on what variables are assumed to be the risks that are priced in the market and which prices of risk are constrained to be zero – the assumptions that have to be made with any affine term structure model. By performing a principal component analysis of the premia I am able to identify economically interpretable factors. Forward premia and expected returns of all bonds are related one-to-one, but the former are more "disaggregated" (in a sense explained in the paper), which makes them a better object for statistical analysis. I find that one factor restriction, which is often imposed on bond risk premia, is more restrictive that can be inferred from regressing excess bond returns on a set of forward rates only.

Second, I try to relate the first two, most important components of the forward premia, to macroeconomic variables. It turns out that the first component is related to equity forecasting variables, such as price-to-earnings ratio and (to a smaller extent) *cay*. Apart from that, it is also related to the dollar index and a measure of investor sentiment. The second component seems to be mostly related to variables describing real activity, such as PMI manufacturing index and capacity utilization plus effective fed funds rate.

Third, I relate the forward premia on the bond market to cross sectional effects of sorting stocks based on various characteristics. I find that only the first principal component is significant in explaining cross sections of equity. I find a non-monotone effect of this component on stock portfolios formed by book-to-market, but monotone patterns for earnings-to-price and size sorts. I also find a puzzling pattern in the momentum-based dimension – both extreme losers and winners have significantly positive sensitivities to the movements in the analyzed component, which however has no effect on more intermediate portfolios.

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