Collateral and Inefficient Continuation

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Abstract

All good firms resemble one another, but each bad firm goes bankrupt in its own way. Empirical studies indicate that many firms are liquidated far too late. This provokes the question: why should any investor decide to finance unprofitable businesses? Potentially there is a number of plausible answers. In this paper I study a strategic connection of inefficient continuation with the price of collateral. The main result is that a bank may decide for inefficient continuation of one company in order to secure repayment of debt from another firm.

1 Introduction

1.1 Empirical motivation

Duffie and Lando (2001) argue that we should accept a certain degree of default unpredictability. Accounting information imperfections have the power to confuse any investor and undermine the accuracy of any model designed to determine the default boundary.

Theoretical corporate finance models recognize two possible causes of default. Defaults, broadly speaking, are triggered either by economic problems or financial distress. Economic distress arises when the firm's prospects deteriorate. Worse prospects are followed by a decrease in the value of business. The standard approach in structural models of credit risk is to assume that default occurs when the market value drops below a certain threshold level. However, those structural models assume that external financing is costless. In contrast, when financing frictions are introduced, a firm may default just because of liquidity problems. Usually this happens to highly levered companies that incur a decline in cash flows. Even a temporary cash crisis of a fundamentally sound business may result in a default.

Empirically the magnitude of economic distress can be measured by how low the market value of assets is relative to debt. In contrast, financial distress is associated with insufficient financial reserves relative to required payments. Davydenko (2007) finds that at default most firms are insolvent both economically and financially. Persistent economic distress depletes cash reserves necessary to pay creditors and suppliers. But these two factors are of a distinct nature. There are examples of economically solvent firms that default due to liquidity problems. On the other hand many companies default although they have high cash reserves. But much more surprising is the significant percentage of low-liquidity and low-value firms that are able to avoid default for years. Even more, Davydenko found that in fact a majority of firms with negative economic net worth do not default for at least a year.

This result seems to be a paradox. Why should any investor finance an unprofitable business? One answer may be provided by a model of Kahl (2002). Kahl emphasizes that very often creditors lack the necessary information that is needed to make a quick and correct liquidation decision. He argues that the long term nature of financial distress is a by-product of a dynamic learning process. Creditors engage in a so-called controlled liquidation. Such a strategy may prove attractive to creditors, because it allows them to participate in the recovery of the firm and receive more than just the liquidation value of assets.

In this paper I present an alternative explanation for inefficient continuation of companies. I hypothesize that inefficient continuation of a firm may occur as a deliberate strategy for the bank even if the bank is perfectly informed about the bad prospects of the company. A bank would not agree to continue non-viable businesses without some good reason. I assume that the bank in its portfolio holds more than one firm linked by the same collateral. Moreover, debt capacity is subject to strict collateral constraints because of limited enforcement. So in case one firm goes bankrupt and its collateral is sold on the market then the value of collateral held by the other firms also drops. In particular, if the price function is steep enough there may be one firm big enough to depress the market price of collateral very significantly. Then the owners of some smaller firms might even prefer to surrender their collateral rather than repay the loan. Obviously, the bank will anticipate such a possibility and must respond adequately. The simplest solution would be to decrease the amount of loans. However, as I will show later, inefficient continuation of the bigger firm may secure repayment of the loans and keep the loans at a high level.

Section 2 provides a description of the problem and a numerical example. Section 3 contains the results.

1.2 Related literature

In general equilibrium models with complete markets, agents keep all their promises. Yet, this assumption does not exactly correspond with reality. Therefore, in models of incomplete markets a default on promises may occur. The simple difficulty with promises is that they require some mechanism to make sure they are kept. This mechanism can take the form of either utility penalties or collateral. There were or still are many popular utility penalties; for instance: imprisonment, hanging, exile to the colonies or debt bondage. Even now, 40 million people in India are bonded workers, many working to pay off debts that were incurred generations ago. Although utility penalties have some charm, in this paper I will concentrate only on the second method, that is, on the collateralized promises.

Geanakoplos and Zame (2002) argue that the reliance on collateral to secure loans has a profound impact on the efficiency of markets, on the allocation of commodities and assets, on their prices and on the volatilities of these prices. In their model, the most important reason for these effects is that collateral is scarce. One important assumption in their model is that there are no penalties to defaulting. As a consequence, every borrower will deliver the minimum of what he owes in every state and the value of collateral he put up to secure his promise. In my model I will use the same assumption.

The motivation for my model comes from empirical studies. But the model presented in this paper was inspired mainly by two papers: Bester (1994) and Shleifer and Vishny (1992). Bester in his model considers the impact of collateral on the probability of debt renegotiation. The main assumption is that the bank is less efficient than a manager of the project's assets and so the bankruptcy is ex post inefficient. Moreover, the bank cannot distinguish whether the borrower defaults voluntarily or is unable to meet his payment. The bank cannot credibly commit to liquidating the defaulting firm either. As a result, the borrower has the incentive to claim falsely that the debt exceeds the return and that he is forced to default. This would force the bank to enter into a debt renegotiation process and perhaps agree to forgive a portion of the debt. The problem can be alleviated by the issuance of debt that is secured by outside assets. Collateral reduces the debtor's motives for voluntary default and so the bankruptcy is less likely to occur. Bester in his paper uses a debt renegotiation process simplified to just two steps, where the borrower can demand debt relief and the bank can either agree or liquidate the firm. I will make use of the same debt renegotiation process but with different assumptions. I assume that the bank has perfect information and that there is limited debt enforcement. Consequently, collateral plays a different role. Most importantly, collateral determines the debt capacity for the firms. But, just like in Shleifer and Vishny (1992), the price of collateral is not fixed.

Shleifer and Vishny (1992) study debt capacity and the choice of optimal leverage in a model with aggregate states. They argue that debt may result in forced liquidations in bad times which in turn may limit the leverage that firms choose. Additionally, asset liquidations in bad times, the so called fire sales, can have substantial welfare costs. Similarly, in my model the bank wants to prevent fire sales of the assets.

2 Initial problem description

The model has three periods: 0, 1 and 2. All the important action takes place at time 1. There are four players: companies A and B, Nature and a bank. At time 0 both companies sign debt contracts with the bank for independent projects. Loans are protected by collateral. The market interest rate is zero. Firms are run by managers who are liquidation averse. They always prefer to continue and they would never voluntarily liquidate the firm. There are five main assumptions:

- profits that the companies generate are not verifiable by the court and so companies cannot be forced to repay their debts
- loans granted by the bank can be secured only with collateral. However, every liquidation of the company and its collateral inevitably depresses the market price of collateral
- the price of collateral is determined endogenously, that is, only by the actions taken by players of the model
- the companies hold exactly the same type of collateral but different amounts
- the managers of the companies have private values for the collateral equal to its price times the number of units

Notation:

L_i - face value of debt for company i

s_i - probability that the first period project is successful

V_i - value of the collateral held by company i

c_i - amount of collateral held by company i

g - probability that the new project at time 1 is profitable

Y_i^1- profit from the first period project

$\overline{Y}_i^2$ - profit from good project at time 2

$\underline{Y}_i^2$ - profit from bad project at time 2

ΔL_i - debt relief for company i

The inverse demand function for collateral is assumed here to be linear

$$p(c) = z - \alpha c$$

This is obviously a very strong assumption, but it will simplify derivations and the results will be easier to interpret. In order to make the notation shorter, let

$$V_A\left(p^A\right) = p^A c_A = \left(z - \alpha c_A\right) c_A$$

denote the value of the smaller firm's collateral when it is liquidated on the market, here p^A denotes the price when collateral of firm A is liquidated. Respectively p^B will denote the price when B is liquidated and p^{A+B} when two are liquidated. Also

$$V_A\left(p^B\right) = p^B c_A = \left(z - \alpha c_B\right) c_A$$

denotes the value of collateral held by firm A after company B is liquidated. In turn,

$$V_A\left(p^{A+B}\right) = p^{A+B} c_A = \left(z - \alpha\left(c_A + c_B\right)\right) c_A$$

denotes the value of firm's A collateral after both the smaller A and the bigger B companies' collateral were sold on the market. Consequently for $c_A < c_B$, it must be true that

$$V_A\left(p^{A+B}\right) < V_A\left(p^B\right) < V_A\left(p^A\right) < V_B\left(p^B\right)$$

The following example provides a short description of the problem.

Example 1 *Assume that the price function of the collateral is equal to*

$$p\left(c\right) = 15 - c_A - c_B$$

Firm A holds 2 units of collateral and B holds 6 units. If only firm A goes bankrupt and is liquidated then the collateral's market price drops to 13 and its value to 26. Consequently firm A's debt capacity is no more than 26. *Firm B is larger and it has a maximal debt capacity equal to 54. From the*

	price of collateral	= 15 - cA - cB	
	company:	A	B
	c - units of collateral	2	6
A liquidated		price =	13
		Va	Vb
	value of collateral	26	78
B liquidated		price =	9
		Va	Vb
	value of collateral	18	54
both are liquidated		price =	7
		Va	Vb
	value of collateral	14	42

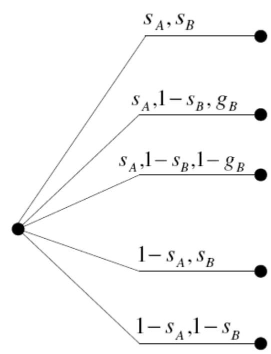
table we can see that if the smaller company is liquidated then B still prefers to repay its debt. That is because the small company A has less impact on the price of collateral. When A's collateral is liquidated then B's collateral has a value equal to 78*, which is still far above the maximal face value of B's debt:* $L_B = 54$*. However, if firm B goes bankrupt and all of its collateral is liquidated then price of collateral drops to 9. Also the value of A's collateral drops to* $V_A\left(p^B\right) = 18$*. The bank to protect itself from the impact of B on A should not lend more than* $L_A = 18$ *to company A and* $L_B = 54$ *to company B. Obviously, the bank can keep its position completely safe by lending only* $L_A = 14$ *and* $L_B = 42$*.*

As the example points out, it is always possible to grant loans at a level that protects the bank against all contingencies. However, this would be a rather extreme protection for the bank.

2.1 Timing

- Time 0:

  Projects start.



States of Nature.

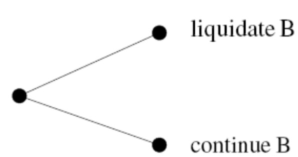
For each company the first project ends at time 1, with a probability of success s_i. For simplicity, Nature will draw the second period project only for the bigger company B. If the bigger company fails, Nature draws another project which is either good or bad. It is good with probability g_B.

- Time1:

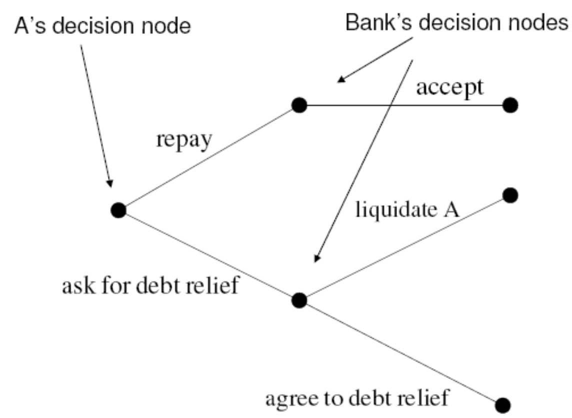
Let us assume that company B failed and A was successful. For the model to be interesting we need one company to be much bigger than the other, big enough to cause a drop in the value of A's collateral below the face value of its debt. Assume also that the bigger company B is obliged to repay its debt before company A. Consequently the bank has some time to react to the failure of company B before A makes its move. In other words, the sequence of moves is the following:

$$B \rightarrow \text{bank} \rightarrow A \rightarrow \text{bank}$$

Now, if company B fails then the bank has one of two choices: either to shut down the company and liquidate its collateral at the market or let it continue until time 2.



Next is the move of company A. The manager of company A may either obediently repay the debt or try to take advantage of B's problems. Liquidation of company B has a negative impact on the value of its collateral. Therefore, if the market value of A's collateral is below the face value of debt, the manager of company A may prefer to surrender its collateral instead of repaying the loan. Equivalently, he might also ask for debt relief. To such a blackmail, the bank can react by either agreeing to debt relief or seizing the collateral and liquidating the company.



- Time 2

Profits from the second project, if it has started, are realized at time 2. Also a company that failed at time 1 can be liquidated either at time 1 or at time 2. The decision when to liquidate the company and its collateral is made by the bank.

2.2 States of Nature and information partitions

There are six states of Nature:

$$\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6\}$$

where μ is the system of prior beliefs:

$$\begin{aligned}
\mu(\omega_1) &= s_A s_B \\
\mu(\omega_2) &= s_A(1-s_B)g_B \\
\mu(\omega_3) &= s_A(1-s_B)(1-g_B) \\
\mu(\omega_4) &= (1-s_A)s_B \\
\mu(\omega_5) &= (1-s_A)(1-s_B)g_B \\
\mu(\omega_6) &= (1-s_A)(1-s_B)(1-g_B)
\end{aligned}$$

So ω_1 corresponds to the state of Nature when both companies were successful with their first project and so on. T is a partition of set Ω. The bank has full information. The managers always know their own type, but they do not observe directly whether the other company was successful or not. In particular, the smaller company does not know whether the bigger firm B was successful or not:

$$T_A = \{\{\omega_1, \omega_2, \omega_3\}, \{\omega_4, \omega_5, \omega_6\}\}$$

and for the bigger firm the type partition is:

$$T_B = \{\{\omega_1, \omega_4\}, \{\omega_2, \omega_5\}, \{\omega_3, \omega_6\}\}$$

All the managers can observe is whether the collateral of the other company was liquidated or not. Liquidation of firm's i collateral implies that the company i was unsuccessful with its project from time 0. However, no liquidation at time 1 does not imply that both companies were successful.

2.3 Payoffs to the smaller company

Payoffs from the project for the firm are not modeled here. So the company concentrates only on minimizing the cost of the credit. If there are no collateral liquidations then the smaller company repays its debt, which amounts to a negative payoff of $-L_A$. However, if the bigger company is liquidated then firm A also incurs the loss of $V_A\left(p^A\right) - V_A\left(p^B\right)$ in the value of its assets used as collateral. Therefore, the smaller company may want to be compensated for this by demanding debt relief. The value of debt relief depends on the bargaining power of the smaller company. But here it will be assumed that firm A demands debt relief equal to or smaller than the value that makes it indifferent between repaying the loan and surrendering collateral:

$$L_A - \Delta L_A \geq V_A\left(p^B\right)$$

2.4 Payoffs to the bank

a) Assume that the smaller company A decides to repay its debt and the unsuccessful company B offers a new project. Then the bank's payoff will be one of the following:

$$\begin{aligned}
\text{if } B \text{ is liquidated} &: \quad L_A + V_B\left(p^B\right) \\
\text{if } B\text{'s good project is accepted} &: \quad L_A + \overline{Y}_B^2 \\
\text{if } B\text{'s bad project is accepted} &: \quad L_A + \underline{Y}_B^2 + V_B\left(p^B\right)
\end{aligned}$$

b) Assume that the smaller firm A refuses to repay its loan and asks for debt relief. At the same time, firm B offers a new project, then the bank's payoff will be one of the following:

$$\begin{aligned}
(dr\text{,continue } B) \text{ and good project} &: \quad L_A - \Delta L_A + \overline{Y}_B^2 \\
(dr\text{,continue } B) \text{ and bad project} &: \quad L_A - \Delta L_A + \underline{Y}_B^2 + V_B\left(p^B\right) \\
(dr\text{,liquidate } B) &: \quad L_A - \Delta L_A + V_B\left(p^B\right) \\
(\text{liquidate } A\text{,continue } B) \text{ and good project} &: \quad V_A\left(p^A\right) + \overline{Y}_B^2 \\
(\text{liquidate } A\text{,continue } B) \text{ and bad project} &: \quad V_{A+B}\left(p^{A+B}\right) + \underline{Y}_B^2 \\
(\text{liquidate } A\text{,liquidate } B) &: \quad V_{A+B}\left(p^{A+B}\right)
\end{aligned}$$

where dr means that the bank agrees to make debt concessions. It will be assumed that a good second period project profit accrues to the bank with a payoff equal to the initial loan, while a bad project has a payoff equal to zero:

$$\begin{aligned} \overline{Y}_B^2 &= L_B \\ \underline{Y}_B^2 &= 0 \end{aligned}$$

2.5 Additional conditions

In order to make the game interesting several conditions must be met:

a) Company A was successful and either B was successful as well or B has a good project for the next period. Company B with its new project will be able to repay the loan in an amount equal to L_B. We would like to make sure that, in this case, the bank will always decide to liquidate company A if it tries to blackmail the bank by asking for debt relief. This will happen if

$$\text{payoff (liquidate } A) \geq \text{payoff (agree to debt relief for } A)$$

and after we plug payoffs we get

$$\Delta L_A + V_A\left(p^A\right) - L_A \geq 0$$

which is always true.

b) We are also interested in cases where the bank, after liquidating company B, will find it more profitable to agree to debt relief for company A if the manager of A asks for it:

$$\text{payoff (}B\text{ is liquidated and }A\text{ receives dr)} \geq \text{payoff (liquidate both companies)}$$

which is true when

$$\Delta L_A + V_A\left(p^A\right) - L_A \leq 2\alpha c_A c_B$$

c) Assume that A was successful and B failed, but it has a good second period project. As was said before, the bank knows all the realizations of nature. In this case, we are interested in a condition securing that B will start its new project. So the bank must be better off if B continues and A repays its loan, rather than from liquidating B and granting debt relief to A:

$$\text{payoff}\,(B \text{ continues and } A \text{ pays back its loan}) \geq \text{payoff}\,(\text{liquidate } B \text{ and agree to dr for } A)$$

which is true if

$$V_B\left(p^B\right) - L_B \leq \Delta L_A$$

d) Finally, we must also make sure that even if the unsuccessful company B has nothing to offer but a bad project, the bank will still prefer to continue B in order to deceive A and secure the repayment of the loan :

$$\text{payoff}\,(B \text{ continues and } A \text{ pays back its loan}) \geq \text{payoff}\,(\text{liquidate } B \text{ and agree to dr for } A)$$

which is always true, because

$$0 \geq -\Delta L_A$$

These conditions also guarantee that the smaller company A will not try to ask for debt relief neither when firm B was successful nor when B failed with its first project but has a new good project.

Minimal debt requirements Later on we will need two more assumptions about the minimal debt requirements. We will assume that the smaller firm A to start the new project needs at least:

$$L_A - V_A\left(p^B\right) \geq 0$$

Moreover, a lower bound for B's face value of debt is needed. It may also be assumed that the bigger company needs a loan equal to at least:

$$L_B^* \geq V_B\left(p^B\right) - \Delta L_A$$

3 Results

3.1 Inefficient continuation

The main point of this section is to demonstrate that inefficient continuation may occur as a deliberate strategy for the bank. The result will be shown in steps. First the difference between the strategies for the smaller and the bigger company will be pointed out. That is, under the assumptions of this model for overcollateralized loans, the bigger company has "no degrees of strategic freedom". The next lemma shows that the bigger company does not respond in any way to a default of the smaller company. So, as far as this model is concerned, all the actions of the bigger company are determined by the realizations of Nature and decisions made by the bank.

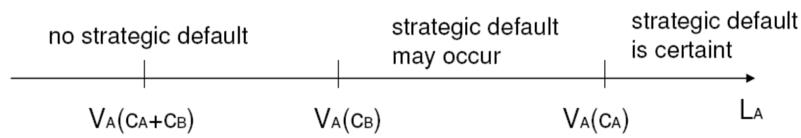
Lemma 2 *Assume that the smaller company went bankrupt and its collateral was liquidated on the market. Then the manager of B will still prefer to repay his debt rather than surrender his collateral.*

Proof. From the assumption of non-verifiable profits, we have $L_B \leq V_B\left(p^B\right)$. But then for $c_A < c_B$ we have

$$L_B \leq V_B\left(p^B\right) < V_B\left(p^A\right)$$

so B will not try to default strategically, because the bank would not miss such an opportunity to liquidate the collateral held by firm B. ■

This is not the case for the smaller company. The credit decisions of firm A's manager will depend on the success or failure of the bigger company. If the bigger company is liquidated then the value of A's collateral drops and the manager of A has to take the decision.



This is summarized in the next lemma.

Lemma 3 *For* $L_A \leq V_A\left(p^A\right)$ *the loan granted to company A is fully protected. If*

$$L_A \in \left[V_A\left(p^{A+B}\right), V_A\left(p^B\right)\right]$$

the bank is secured against any strategic default attempts of the smaller company. Here the bank might lose only if both companies fail at the same time. In the following region the bank is vulnerable to strategic default by the smaller company:

$$L_A \in \left[V_A\left(p^B\right), V_A\left(p^A\right)\right]$$

Finally, above $V_A\left(p^A\right)$ *no loan will be granted.*

Proof. If $L_A \leq V_A\left(p^A\right)$, then the bank is secured against all contingencies. Even if A fails together with B, then the bank cannot lose anything on the loan to A because it is fully covered. If L_A is above $V_A\left(p^A\right)$ but still below $V_A\left(p^B\right)$, then the bank might incur some losses from a loan to A only if both companies fail. But what is important, for $L_A \leq V_A\left(p^B\right)$ no strategic default will take place as the value of collateral cannot drop below the face value of debt. After the bigger company fails and has no good prospects for the next period, it is optimal for the bank to liquidate it. However, if $L_A \in \left[V_A\left(p^B\right), V_A\left(p^A\right)\right]$ and B is liquidated, then the manager of the smaller company will refuse to repay his debt, because $L_A > V_A\left(p^B\right)$. That is, the value of his collateral dropped to a level where it is more profitable to surrender the collateral to the bank. ■

The above lemma shows that the smaller company, for some values of debt, may try to blackmail the bank for debt relief. Obviously, such prospects must influence the bank's willingness to grant any loan at all. The next two lemmata pinpoint the participation constraints for the bank. First, it

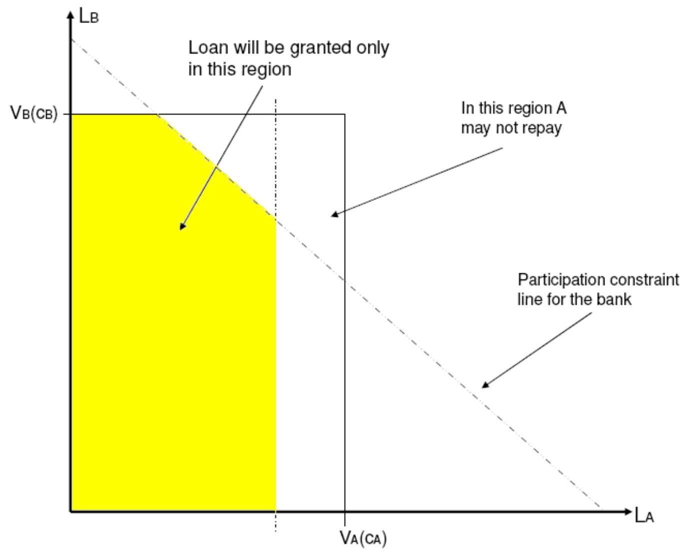


Figure 1: Feasible combinations for loans.

is assumed that the smaller company will repay. Based on this, an inequality connecting the values of L_A, L_B and the probabilities is identified. Then in the next step one more constraint is added, which ensures repayment of debt by the smaller company.

Lemma 4 *Assume that the smaller company always repays its debt. Then under the assumption of a zero interest rate the following condition must be satisfied in order to make sure that the bank will be willing to participate in the game:*

$$L_B \leq \theta\tau L_A + \tau\Phi$$

where

$$\begin{aligned}\theta &= s_A s_B + s_A(1-s_B) - 1\\ \tau &= \frac{1}{1-(1-s_A)s_B - s_A s_B - (1-s_A)(1-s_B)g_B}\\ \Phi &= [(1-s_A)s_B + (1-s_A)(1-s_B)g_B]\,V_A\left(p^A\right) + s_A(1-s_B)V_B\left(p^B\right)\\ &\quad + (1-s_A)(1-s_B)(1-g_B)V_{A+B}\left(p^{A+B}\right)\end{aligned}$$

Proof. Bank will not grant any loan unless its payoff is at least non-negative:

$$\begin{aligned}& s_A s_B(L_A+L_B) + (1-s_A)s_B\left\{V_A\left(p^A\right)+L_B\right\} + s_A(1-s_B)\left\{L_A + V_B\left(p^B\right)\right\}\\ & + (1-s_A)(1-s_B)g_B\left\{V_A\left(p^A\right)+L_B\right\} + (1-s_A)(1-s_B)(1-g_B)V_{A+B}\left(p^{A+B}\right)\\ \geq\ & L_A + L_B\end{aligned}$$

and after some manipulation we get the condition from the lemma. ■

Next, it is necessary to find conditions under which the manager of the smaller company A will repay his debt if he can. In other words, the bank has to decide for which combinations of (L_A, L_B) the manager of company A will repay his debt if he does not observe any collateral liquidation.

Lemma 5 *The manager of the smaller company will always repay his debt if he does not observe any liquidation of the collateral at the market and there is the following upper bound on the amount of his debt:*

$$L_A \leq V_A\left(p^A\right) - \frac{(1-s_B)(1-g_B)}{s_B + (1-s_B)g_B}\Delta L_A$$

Proof. In subsection 2.5 conditions were stated for which the manager of company A knows when he should not ask for debt relief. All A can observe is whether collateral that belonged to B was liquidated on the market or not. However, the problem is that he cannot observe whether B was successful or not, neither does he know the quality of the bigger firm's new project. But A knows the probabilities of particular events. For that lemma to be true, the bank needs sufficient means to prevent the smaller company from strategically defaulting. In other words, it must be feasible for the bank to convince the manager of the smaller company that he should better repay his debt. A will repay if the expected loss from strategic default does not exceed the expected loss from repaying the debt:

$$\begin{aligned} & s_B \{-L_A\} + (1-s_B)(g_B)\{-L_A\} + (1-s_B)(g_B)\left\{-L_A - \left[V_A\left(p^A\right) - V_A\left(p^B\right)\right]\right\} \\ \geq\; & s_B\left\{-V_A\left(p^A\right)\right\} + (1-s_B)(g_B)\left\{-V_A\left(p^A\right)\right\} \\ & + (1-s_B)(g_B)\left\{-L_A + \Delta L_A + \left[V_A\left(p^A\right) - V_A\left(p^B\right)\right]\right\} \end{aligned}$$

and rearranging we arrive at

$$L_A \leq V_A\left(p^A\right) - \frac{(1-s_B)(1-g_B)}{[s_B + (1-s_B)(g_B)]}\Delta L_A$$

This is the condition necessary to secure repayment from the smaller company, if its manager is

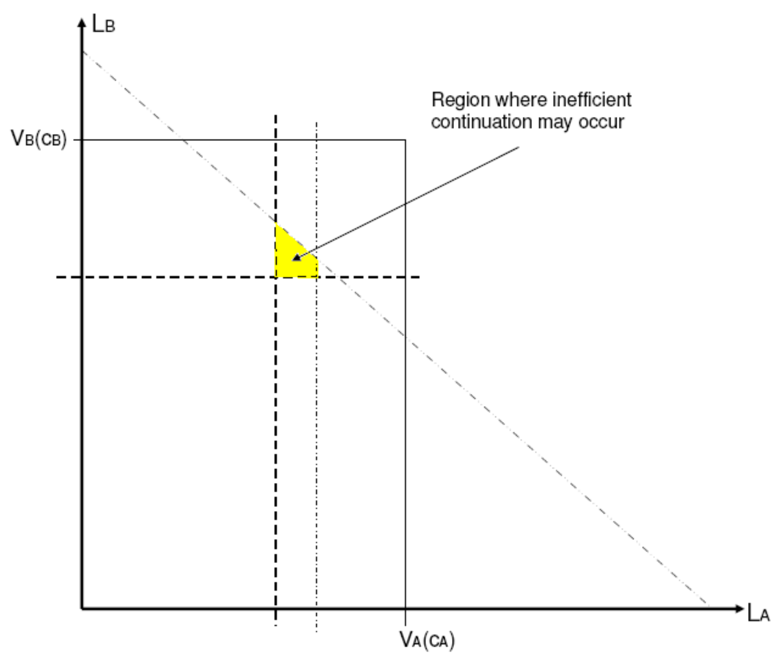


Figure 2: The inefficient continuation region.

unsure whether the bigger company defaulted or not. ■

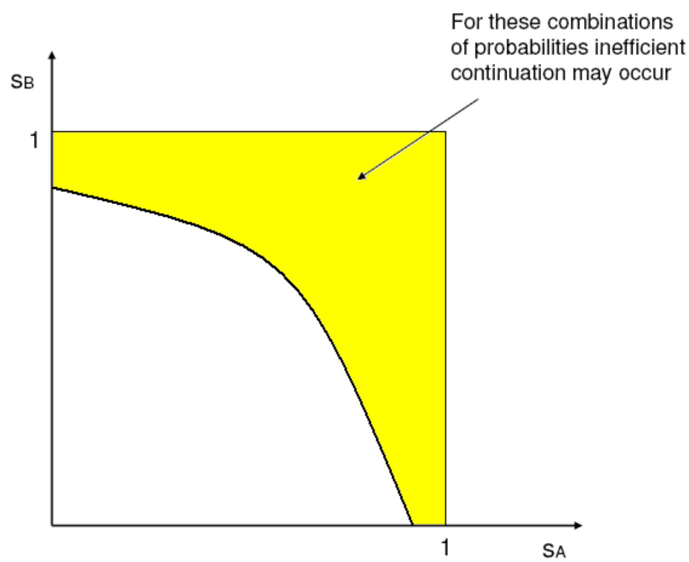


Figure 3: Probabilities supporting the inefficient continuation region.

Figure 1 depicts the conditions from the above lemmata. First, the face value of debt cannot exceed the value of the collateral when it is liquidated, here these are $V_A\left(p^A\right)$ and $V_B\left(p^B\right)$. Second, the participation constraint, from Lemma 4, for the bank has to be satisfied. Finally, the face value of debt for the smaller company must not exceed the upper boundary from Lemma 5. The yellow region in the graph illustrates all combinations of L_A and L_B that are acceptable for the bank. It is quite likely that the companies cannot start their projects without some prespecified amount of investment. If it happens that the two companies need respectively:

$$L_A > V_A\left(p^B\right) = \left(z - \alpha c_B\right) c_A$$

$$L_B > V_B\left(p^B\right) - \Delta L_A = \left(z - \alpha c_B\right) c_B - \Delta L_A$$

then inefficient continuation may occur.

Proposition 6 *Assume that the bigger company failed with its time 0 project. If the face value of* *debt granted to both firms is sufficiently high, then for some combinations of probabilities* (s_A, s_B, g_B), *inefficient continuation of the bigger company may occur if*

$$\begin{aligned} L_A^* &\geq V_A\left(p^B\right) \\ L_B^* &\geq V_B\left(p^B\right) - \Delta L_A \\ s_B &\geq \frac{\Upsilon}{\Psi} \end{aligned}$$

where

$$\begin{aligned} \Upsilon &= V_A\left(p^A\right) + V_B\left(p^B\right) - 2\Delta L_A - \left(s_A + (1 - s_A)\, g_B\right)\left(V_A\left(p^A\right) + V_B\left(p^B\right) - \Delta L_A\right) \\ &\quad - (1 - s_A)\,(1 - g_B)\, V_{A+B}\left(p^{A+B}\right) \end{aligned}$$

and

$$\begin{aligned} \Psi &= V_A\left(p^A\right) + V_B\left(p^B\right) - \Delta L_A - s_A\left(V_A\left(p^A\right) + V_B\left(p^B\right)\right) \\ &\quad + (1 - s_A)\, g_B\left(V_A\left(p^A\right) + V_B\left(p^B\right) - \Delta L_A\right) - (1 - s_A)\,(1 - g_B)\, V_{A+B}\left(p^{A+B}\right) \end{aligned}$$

Proof. The conditions from section 2 guarantee that the bank's payoff is higher when A repays its loan and the bigger company is liquidated one period later, rather than agreeing to debt relief for A and liquidating B immediately. The smaller firm may ask for debt relief only if the value of collateral dropped below the face value of debt. That is, after B was liquidated, the difference between the face value of debt and the value of collateral is above zero. So we need to use now the minimal debt requirements

$$\begin{aligned} L_A^* &\geq V_A\left(p^B\right) \\ L_B^* &\geq V_B\left(p^B\right) - \Delta L_A \end{aligned}$$

However, if at time 1 the big company fails then for a such high L_B, the bank will have to agree to debt relief ΔL_A. Liquidating both firms at the same time would result in a bigger loss than the liquidation of just B and debt relief for A. So once the lower bounds for loans are fixed it is necessary to check whether the conditions from Lemma 4 and 5 are satisfied. This situation is depicted in Figure 2. Inefficient continuation may occur in the yellow region. But this region may be empty if all combinations of L_A^* and L_B^* are above the participation line for the bank. In order to check whether the participation constraint is not violated it is enough to plug $L_A^* = V_A\left(p^B\right)$ into

$$L_B \leq \theta\tau L_A + \tau\Phi$$

then solve for L_B and finally compare this value with $L_B^* = V_B\left(p^B\right) - \Delta L_A$. After some manipulations, we get the condition that links the probabilities:

$$s_B > \frac{\Upsilon}{\Psi}$$

If this is satisfied then inefficient continuation may occur. ■

Figure 2 presents the region where inefficient continuation might occur. Figure 3 illustrates all the combinations of (s_A, s_B) that support the region of inefficient continuation. So if all the conditions from the above propositions are satisfied and (s_A, s_B) belongs to the yellow region then inefficient continuation occurs. The region where the inequality $s_B > \frac{\Upsilon}{\Psi}$ is true depends also on the value of g_B. It turns out that the higher the value of g_B the larger also the volume of the feasible region. So for high values of g_B, not surprisingly a bigger volume of s_A and s_B combinations can support the participation constraint. Figure 4 depicts the difference between the feasible regions for small and high values of g_B.

3.2 Welfare analysis

At the beginning we assumed that the bad second period project of the bigger firm brings the payoff of $\underline{Y}_B^2 = 0$. However, we may suspect that in many cases the bank would need to incur some

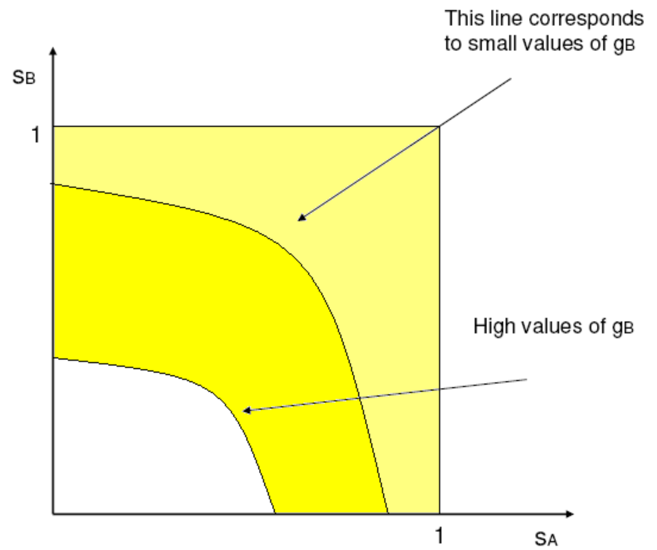


Figure 4: Small and high realizations of g.

additional costs in order to let the bigger firm B survive for one period more. Let us call these additional costs ε; this additional cost corresponds to the negative NPV from the second period project. Inefficient continuation may still occur but a volume of the region where it does will shrink. This is summarized in the next lemma.

Lemma 7 *If the second period project for the bigger firm B has negative NPV, that is $\underline{Y}_B^2 = \varepsilon < 0$, then inefficient continuation may still occur if*

$$\begin{aligned} L_A^* &\geq V_A\left(p^B\right) \\ L_B^* &\geq V_B\left(p^B\right) - \Delta L_A \\ s_B &\geq \frac{\Upsilon^{\varepsilon}}{\Psi^{\varepsilon}} \end{aligned}$$

where

$$\Upsilon^{\varepsilon} = \Upsilon - s_A (1 - g_B) \varepsilon$$
$$\Psi^{\varepsilon} = \Psi + s_A (1 - g_B) \varepsilon$$

Proof. It is easy to observe that the participation constraint line for the bank will be shifted downward

$$L_B \leq \theta \tau L_A + \tau \Phi + s_A (1 - s_B) (1 - g_B) \varepsilon$$

where θ, τ and Φ are the same as in Lemma 4. Now, using the new participation constraint we proceed like in the proof of Proposition 6. After some manipulations, we get the result. ■

Definition 8 *Social welfare is equal here to the sum of the expected profits for the bank and both firms.*

Using this lemma we can now say something about the welfare consequences of inefficient continuation.

Proposition 9 *The possibility of using inefficient continuation of firm A is ex ante welfare improving, because for*

$$L_A^* \geq V_A \left(p^B\right)$$
$$L_B^* \geq V_B \left(p^B\right) - \Delta L_A$$

the bank would not issue any credit. This would result in an expected loss in the social welfare equal to

$$\textit{social welfare loss} = s_A \left(Y_A^1 - L_A\right) - (1 - s_A) V_A \left(p^A\right)$$

However, ex post for $\varepsilon < 0$ inefficient continuation of firm B brings loss equal to ε.

Proof. At time 0, the bank will not issue any loans above the level of L_A^* and L_B^* if it cannot rely on inefficient continuation of the bigger firm to secure repayment from A. But if for some reason inefficient continuation is impossible, then the smaller firm A will not start any project with the necessary investment above L_A^*. The bigger firm B will be able to start its project, because this firm always repays as was pointed out in Lemma 2. Social welfare would then incur a loss equal to the sum of expected profits for the smaller firm A and the bank. Now, since the interest rate was assumed to be zero then the expected profit of the bank is also equal to zero.

At time 1, after the bigger firm failed and has only bad quality second period project the bank will decide for inefficient continuation in order to secure repayment of the loan from the smaller firm. Firm A at time 1 can gain on debt renegotiation an amount equal to ΔL_A which would be also equal to the loss of the bank. So from the social welfare point of view this exchange is immaterial. But if the bank decides to incur the cost of ε then inefficient continuation is harmful to the welfare. ■

4 Conclusions

In this paper an indirect link between the price of collateral, repayment of debt and inefficient continuation is studied. The main result of the paper shows that inefficient continuation of firms may arise as a deliberate strategy for the financing bank. Assuming that debt enforcement is limited and firms hold the same type of collateral, the bank may be forced to postpone liquidation of one firm in order not to jeopardize the repayment of debt from the other firm.

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5 Appendix

Proposition 10 (F, T, v, μ) *is a three player Bayesian game with asymmetric information. A solution exists in pure strategies.*

Proof. First, we need to specify the strategies for each player. All the action happens at time 1, the order of moves is the following: B →bank→ A →bank. The bigger firm B moves first. Its moves are determined by Nature if it fails with the first period project. But if firm B was successful then it has just one information set and two choices: repay the debt or ask for debt relief. The manger of firm A, after his firm was successful, has just two information sets. He either observes liquidation of the bigger firm B or he does not and then adequately decides either to repay the loan or ask for debt relief. Let a^s denote a vector of information sets for A. Then

$$a^s = (a_1^s,\, a_2^s)$$

and assume that in the information set a_1^s the manager of firm A observed the liquidation of firm B, while in the information set a_2^s the manager of A makes his decision after he observed the liquidation of company B. The smaller firm A can choose between 4 different pure strategies.

Finally, the bank has a lot more pure strategies because it has altogether nine information sets. Let b^s denote a vector of information sets for the bank,

$$b^s=\left(\underbrace{\overbrace{\underbrace{b_1^s}_{\substack{A\text{ was}\\ \text{successful}}}\;,\;\underbrace{b_2^s}_{\substack{A\\ \text{failed}}}}^{\substack{B\text{ was}\\ \text{successful}}}\overbrace{\underbrace{b_3^s}_{\substack{\text{good new}\\ \text{project}}}\;\underbrace{b_4^s}_{\substack{\text{bad new}\\ \text{project}}}}^{\substack{\text{the first period}\\ \text{project for }B\text{ failed}}}}_{\substack{\text{the bank decides}\\ \text{what to do with }B}}\underbrace{\underbrace{b_5^s}_{\substack{B\text{ was}\\ \text{successful}}}\;\overbrace{\overbrace{\underbrace{b_6^s}_{\substack{B\text{ was}\\ \text{liquidated}}}\;\underbrace{b_7^s}_{\substack{B\\ \text{continues}}}}^{\substack{\text{good new}\\ \text{project}}}\overbrace{\underbrace{b_8^s}_{\substack{B\text{ was}\\ \text{liquidated}}}\;\underbrace{b_9^s}_{\substack{B\\ \text{continues}}}}^{\substack{\text{bad new}\\ \text{project}}}}^{\substack{\text{the first period}\\ \text{project for }B\text{ failed}}}}_{\substack{\text{the bank decides}\\ \text{what to do with }A}}\right)$$

Now, by the nature of the debt contract the bank is obliged to accept every debt repayment. So whenever a firm repays its debt then the bank effectively has 'no choice', because the set of its choices is a singleton set and it is a trivial choice [6]. In the information sets b_1^s and b_2^s the bigger firm B decides to ask for debt relief and the bank can choose either to liquidate it or agree to debt relief.

In the information sets b_3^s and b_4^s the bank has to decide whether to liquidate firm B or let it continue with the next period project. Here b_4^s stands for the decision between efficient liquidation and inefficient continuation. In the sets b_5^s, b_6^s, b_7^s, b_8^s, and b_9^s the bank was asked by firm A for debt relief and now the bank can either agree or liquidate A.

Altogether, the bank has 2^9 different pure strategies, and in total the set of all pure strategy combinations is equal to 2^{12}. The major obstacle in proving the existence of Nash equilibrium in pure strategies is the number of pure strategies for the bank. Fortunately, we may use the lemmata proven above to show that most of the information sets will not be even reached.

First, assume that the bank granted two loans L_A and L_B. I will organize the proof with respect to the bank's information sets.

b_1^s, b_2^s) In the information sets b_1^s, b_2^s the bank has to decide what to do with firm B after the manager of B makes its move. Fortunately the decision of firm B is very easy to predict. From Lemma 2 we know that the manager of firm B will repay whenever he can, because this strategy strictly dominates asking for debt forgiveness. Since it is a debt contract, the bank can only accept the repayment. So we are done with the bank's information sets b_1^s, b_2^s that refer to decisions after the firm B was successful. The information sets b_1^s, b_2^s will not be reached.

b_3^s) If company B fails with its first period project then its future depends on the draws of Nature and the bank's decisions. In the information set b_3^s the bank has to make a decision what to do with B after firm B is endowed with a good new project. Here the bank has the right to liquidate B even though its new project is profitable. The bank will not liquidate B only if it brings a better payoff or a smaller loss. Let us first consider the case where the bank decides to liquidate firm B at b_3^s. Then the collateral belonging to B has to be sold on the market, which is observed by the manger of A. But from Lemma 3 we know that the manager of A in the a_2^s information set will always default and ask for debt relief. Still, the bank may prefer to liquidate B and agree to debt relief for the manager of A. This would happen if the inequality from section 2.5 (b) is not satisfied. In other words, if

$$\Delta L_A + V_A\left(p^A\right) - L_A > 2\alpha c_A c_B$$

then the loss on debt relief to firm A can be more than compensated with the profit from the liquidation of firm B. After B is liquidated then A always asks for debt relief and the bank always agrees. That is, because whenever the following inequality holds

$$L_A - \Delta L_A + V_B\left(p^B\right) > V_{A+B}\left(p^{A+B}\right)$$

it is much better for the bank to agree to debt relief rather than liquidate both firms at the same time. Under the assumptions in the paper this inequality always holds, which follows from

$$L_A - \Delta L_A \geq V_A\left(p^B\right)$$

and

$$V_{A+B}\left(p^{A+B}\right) = V_A\left(p^B\right) + V_B\left(p^B\right) - \alpha c_A\left(c_A + c_B\right)$$

and plugging it back we get

$$0 > -\alpha\left(c_A + c_B\right)$$

So the bank always has to agree to debt relief for A after firm B was liquidated. To sum up, after firm B with a good project was liquidated then the manager of A will refuse to pay. This decision is followed by the b_6^s information set.

If, on the other hand, the condition from section 2.5 (b) is satisfied and the bank does not liquidate firm B then the manager of A in the next move finds himself in the information set a_1^s. Here the manager of A cannot infer anything about the condition of the bigger company B and he has to take the decision whether to repay or ask for debt forgiveness. Now, if L_A is below the value from Lemma 5 then we know that the manager of A repays its loan. The firm A's decision to repay its debt leaves the bank with just a trivial choice, that is to accept the repayment and the game is finished. The b_7^s information set will not be reached, because the manager of A repaid his debt in the previous move. So if the above mentioned conditions are met then the bank allows B to continue with a good project and the manager of A repays its loan.

b_4^s) Now, let us consider the bank's information set b_4^s, where firm B has only a bad second period project. Here the bank has to choose between efficient liquidation and inefficient continuation of company B.

If the conditions from Lemma 7 are satisfied then the bank continues inefficiently firm B till period 2. This way the bank hides the true condition of B from the manager of A. Consequently, the manager of A is in the a_1^s information set and he repays his loan then the bank accepts the repayment and the game ends.

On the other hand if any of the conditions from Lemma 7 is not satisfied then inefficient continuation does not occur and company B is liquidated. Then the manager of A is in the information set a_1^s, after the liquidation of company B, his best response is to ask the bank for debt relief. So after the manager of A asks for debt relief then the bank is in the b_8^s information set. However, here the choice is simple. The bank will always agree to debt relief for A after firm B was liquidated.

Now, we have to consider the bank's information sets from b_5^s to b_9^s where the bank has to decide what to do with firm A.

b_5^s) The information set b_5^s follows the decision of firm A to ask for debt relief in its a_1^s information set. But we already know that under the conditions from Lemma 5 it is not optimal for the manager of firm A to ask for debt relief when he did not observe the liquidation of firm B. So the information set b_5^s will not be reached.

b_6^s) The information set b_6^s will not be reached if the inequality from section 2.5 (b) is satisfied.

b_7^s) The information set b_7^s is not reached due to Lemma 5.

b_8^s) As it was shown above this information set follows the decision of the bank to liquidate firm B with a bad project and the refusal of the smaller firm A to repay its debt. Here the bank must agree to debt relief for A. However, this information set will not be reached if all of the conditions from Lemma 7 are satisfied.

b_9^s) This information set is not reached because of Lemma 5.

Finally, it is necessary to mention that there is a continuum of L_A and L_B combinations where the companies have some minimal debt requirements that in expectation would bring a loss to the bank. For those loan combinations in the Nash equilibrium the bank will not issue any loan. Assuming that all the conditions from the above lemmata are satisfied, the bank grants the loans to both firms and the Nash equilibrium in pure strategies exists. But a violation of any of those conditions would typically lead either to a smaller loan for the firm A or no loan at all. ■