Improving Portfolio Selection Using Option-Implied Volatility and Skewness*

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Abstract

Our objective in this paper is to demonstrate empirically how one can use option-implied information to improve mean-variance portfolio selection with a large number of stocks, and to document which aspects of option-implied information are useful for improving the out-of-sample performance of mean-variance portfolios. To calculate the optimal mean-variance portfolio weights, one needs to estimate for each stock its volatility, correlations with all other stocks, and expected return. Our empirical evidence shows that, while using the option-implied volatilities and correlations does *not* improve significantly the portfolio variance, Sharpe ratio, and certainty-equivalent return, exploiting information about expected returns that is contained in the volatility risk premium and option-implied skewness increases substantially Sharpe ratios and certainty-equivalent returns, but this is accompanied by higher portfolio turnover.

Keywords: mean variance, option-implied volatility, variance risk premium, option-implied skewness, portfolio optimization

JEL: G11, G12, G13, G17

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1 Introduction

To implement the static mean-variance Markowitz (1952) model in practice, one needs to estimate the means, volatilities, and correlations of stock returns. Traditionally, historical returns data have been used for this estimation, but researchers have found that portfolios based on sample estimates perform poorly out of sample.[1] Several approaches have been proposed in the literature for improving the performance of portfolios based on *historical data*.[2]

In this paper, instead of trying to improve the quality of the moments estimated from historical data, we use the *forward-looking* option-implied moments of the stock-return distribution. The main contribution of our work is to demonstrate empirically *how* one can use option-implied information to improve portfolio selection with a large number of stocks, and to document *which aspects* of option-implied information are particularly useful. Specifically, we study how one can use option-implied volatilities, correlations, skewness, and the volatility risk premium to adjust the volatilities, correlations, and expected returns of stocks in order to improve the out-of-sample performance of portfolios. We find that the improvement in portfolio performance from using option-implied volatilities and correlations is small, contrary to what one may have expected. However, the use of option-implied skewness and the volatility risk premium to adjust expected returns can lead to substantial improvements in Sharpe ratios and certainty-equivalent returns (even when short sales are constrained), but this is accompanied by an increase in portfolio turnover. Our analysis is carried out in a comprehensive fashion: we consider four benchmark strategies ($1/N$, sample-covariance-based minimum-variance, short-sale-constrained minimum-variance, and minimum-variance with shrinkage of the covariance matrix); four performance metrics (portfolio volatility, Sharpe ratio, certainty-equivalent return, and turnover); two data sets (with 100 assets and with 561 assets); two data frequencies (daily and intraday); two portfolio rebalancing periods (daily and monthly); and, two objective functions (mean-variance optimization and utility maximization).

To determine mean-variance portfolio weights, one needs to estimate for each stock its (i) volatility, (ii) correlation with all other stocks, and (iii) expected return. We therefore undertake our analysis in three corresponding steps. In step one, we determine the optimal portfolio using volatilities implied by option prices. In step two, we find the optimal portfolio using correlations implied by option prices. In the first two steps, because of the extensive results in the literature about the poor performance of portfolios that rely on sample estimates

[1]For evidence of this poor performance, see DeMiguel, Garlappi, and Uppal (2009) and the references therein.

[2]These approaches include: imposing a factor structure on returns (Chan, Karceski, and Lakonishok, 1999; MacKinlay and Pástor, 2000), using daily data rather than monthly data (Jagannathan and Ma, 2003), using Bayesian methods (Jobson, Korkie, and Ratti, 1979; Jorion, 1986; Pástor, 2000; Pástor and Stambaugh, 2000; Ledoit and Wolf, 2004b), constraining shortsales (Jagannathan and Ma, 2003), constraining the norm of the vector of portfolio weights (DeMiguel, Garlappi, and Uppal, 2009), and using stock-return characteristics such as size, momentum, and the book-to-market ratio (Brandt, Santa-Clara, and Valkanov, 2009).

of expected returns, we set expected returns to be equal across all stocks.[3] In step three, we find the optimal portfolio when expected returns are scaled based on information implied in option prices. We describe the findings from these three steps below.

In the first step, we find that using the implied volatilities to compute the optimal portfolio does *not* lead to a substantial reduction in the out-of-sample portfolio volatility (standard deviation) or to an increase in the Sharpe ratio and certainty equivalent return. This is surprising because there is a large literature that documents that implied volatilities can predict the stock-return volatility better than sample volatility (see, for example, Blair, Poon, and Taylor (2001) and Jiang and Tian (2005)). We explain that there are two reasons for this. First, the implied volatilities are estimators with large variances because they are based exclusively on current option prices. Second, because the implied volatilities estimate the risk-neutral volatilities, they are biased estimators of the real-world (objective) volatilities, with the gap between the two being the volatility risk premium, as explained in Chernov (2007). However, we find that even the portfolios based on the risk-premium-corrected model-free implied volatilities attain an out-of-sample portfolio volatility that is only about 5% lower than the traditional portfolios based on the historical stock-return data, while the improvement in Sharpe ratio is still insignificant.

In the second step, we find that the benefits from using option-implied correlations are even smaller than the gains from using option-implied volatilities. To understand the reason for this, note that the covariance matrix that improves portfolio performance will be the one that contains enough information about future covariances *and* is stable (gives a small condition number and, correspondingly, less volatile portfolio weights). Our empirical results indicate that, while option-implied volatilities and correlations are better than their historical counterparts at forecasting the future realizations of these moments, the gains are not substantial enough to offset the loss from the increased instability of the covariance matrix, the effect of which appears in the much higher portfolio turnover.

Finally, in the third step, we study how two sources of option-implied information can be used to improve estimates of expected returns. The first is the historical volatility risk premium, and its choice is motivated by the empirical regularity documented by Goyal and Saretto (2009) that assets with high volatility risk premium tend to outperform those with low volatility risk premium. Our empirical evidence shows that portfolios based on expected returns scaled by the volatility risk premium outperform traditional portfolios. The second source of

[3] It is well known that it is much more difficult to estimate expected returns than second moments of stock returns (Merton, 1980), and as a result, much recent research has focused on minimum-variance portfolios, which rely solely on estimates of covariances. In fact, Jagannathan and Ma (2003, pp. 1652–3) write that: "The estimation error in the sample mean is so large nothing much is lost in ignoring the mean altogether when no further information about the population mean is available. For example, the global minimum variance portfolio has as large an out-of-sample Sharpe ratio as other efficient portfolios when past historical average returns are used as proxies for expected returns."

information is option-implied skewness, whose choice is motivated by the finding in Rehman and Vilkov (2008) that stocks with high option-implied skewness outperform stocks with low option-implied skewness.[4] We find that portfolios that use expected returns scaled by implied skewness achieve significantly higher Sharpe ratios than those of traditional portfolios (even in the presence of short-sale constraints), but these gains are accompanied with higher portfolio turnover.

We conclude this introduction by discussing the relation of our work to the existing literature. The idea that option prices contain information about future asset returns has been understood ever since the work of Black and Scholes (1972) and Merton (1973). For example, Latane and Rendleman (1976), Lamoureux and Lastrapes (1993), and Christensen and Prabhala (1998) find that implied volatility outperforms historical volatility in forecasting future volatility, and Poon and Granger (2005) provide a comprehensive survey of this literature. Bakshi, Kapadia, and Madan (2003) explain how one can use option prices to infer also higher moments of the return distribution, such as skewness. Driessen, Maenhout, and Vilkov (2009) show, in the working paper version of their article, how one can obtain also implied correlations from the prices of options on individual stocks and on the index, while Bollerslev, Tauchen, and Zhou (2008), Cremers and Weinbaum (2008), Goyal and Saretto (2009), Rehman and Vilkov (2008), and Xing, Zhang, and Zhao (2009) show that options can also be used to forecast future returns of the underlying asset. Of course, one can extract not just particular moments of returns, but also the probability distribution function, as shown by Jackwerth and Rubinstein (1996), Aït-Sahalia and Lo (1998), Jackwerth (2000), Bliss and Panigirtzoglou (2004), Panigirtzoglou and Skiadopoulos (2004), and Benzoni (1998), while Chernov and Ghysels (2000) show how to estimate jointly both the objective measure and the risk-neutral measure.

The focus of our work is to investigate how the information implied in option prices, which is discussed in the paragraph above, can be used to improve portfolio selection. There are two other papers that study this. The first, by Aït-Sahalia and Brandt (2008), uses option-implied state prices to solve for the intertemporal consumption and portfolio choice problem, using the Cox and Huang (1989) martingale representation formulation, rather than the Merton (1971) dynamic-programming formulation. This paper finds that optimal consumption and portfolio rules based on option-implied information are different from those obtained using standard return dynamics; however, its focus is not on finding the optimal portfolio with superior out-of-sample performance. The second, which is by Kostakis, Panigirtzoglou, and Skiadopoulos (2009), studies the market-timing problem of allocating wealth between the S&P500 index and a riskless asset. The paper uses options on the index to first back out the implied risk-neutral

[4]For the relation between expected stock returns and skewness measured directly, as opposed to option-implied skewness, see Rubinstein (1973), Kraus and Litzenberger (1976), Harvey and Siddique (2000), and Boyer, Mitton, and Vorkink (2009).

distribution of returns and then transforms this to the objective distribution. This paper finds that the out-of-sample performance of the portfolio based on this distribution is better than that of a portfolio based on the historical distribution. However, there is an important difference between this work and ours: rather than considering the *market-timing* problem of how to allocate wealth between the S&P500 index and the riskfree asset, we consider the *portfolio-selection* problem of allocating wealth across a large number of individual stocks; in particular, we consider portfolios with 100 stocks and 561 stocks. It is not clear how one would extend the methodology of Kostakis, Panigirtzoglou, and Skiadopoulos (2009) to accommodate a large number of risky assets.[5]

The rest of the paper is divided into a number of short distinct sections in order to help the reader understand each step of our analysis. In Section 2, we provide a brief background to portfolio selection. In Section 3, we describe the data on stock returns and options that we use in our empirical work. In Section 4, we explain the various measures we use to evaluate portfolio performance. The construction and performance of our benchmark portfolio strategies that do *not* use option-implied information is described in Section 5. How we compute the quantities implied by option prices that we use for portfolio selection is explained in Section 6. Our main findings about the performance of various portfolio strategies that use option-implied information are given in Section 7. The robustness checks we undertake are described in Section 8, and we conclude in Section 9. Appendix A explains how to compute variances and covariances for high-frequency intraday data; Appendix B explains the method used for the shrinkage and regularization of the covariance matrix; and, Appendix C explains the construction of model-free option-implied moments.

2 Portfolio Selection

The classic mean-variance optimization problem is

$$\min_{w} \quad w^\top \hat{\Sigma} w - \frac{1}{\gamma} w^\top \hat{\mu}, \tag{1}$$

$$\text{s.t.} \quad w^\top e = 1, \tag{2}$$

where $w \in \mathbb{R}^N$ is the vector of portfolio weights invested in stocks, $\hat{\Sigma} \in \mathbb{R}^{N\times N}$ is the estimated covariance matrix, $\hat{\mu} \in \mathbb{R}^N$ is the estimated vector of expected returns, and $e \in \mathbb{R}^N$ is the vector of ones. The objective in (1) is to minimize the difference between the variance of the portfolio return, $w^\top \hat{\Sigma} w$, and its mean, $w^\top \hat{\mu}$, after taking into account the risk aversion of the agent, denoted by γ. The constraint $w^\top e = 1$ in (2) ensures that the portfolio weights sum to

[5]Kostakis, Panigirtzoglou, and Skiadopoulos (2009) also need to make other restrictive assumptions, such as the existence of a representative investor and that financial markets are complete.

one. We consider the case without the risk-free asset because our objective is to explore how to use option-implied information to select the portfolio of risky stocks.

The solution to the *mean*-variance problem in (1)–(2) gives the weights in the portfolio of only-risky stocks:

$$w = \frac{\hat{\Sigma}^{-1}\hat{\mu}}{e^{\top}\hat{\Sigma}^{-1}\hat{\mu}}, \tag{3}$$

in which the covariance matrix $\hat{\Sigma}$ can be decomposed into volatility and correlation matrices,

$$\hat{\Sigma} = \mathrm{diag}(\hat{\sigma})\, \hat{\Omega}\ \mathrm{diag}(\hat{\sigma}), \tag{4}$$

where diag $(\hat{\sigma})$ denotes the diagonal matrix with volatilities of the stocks on the diagonal, and $\hat{\Omega}$ is the correlation matrix. Thus, to obtain the optimal portfolio weights in (3) there are three quantities that need to be estimated – expected returns $(\hat{\mu})$, volatilities $(\hat{\sigma})$, and correlations $(\hat{\Omega})$. Information from prices of options can be used to inform our choices of all three quantities.

Given the finding in Jagannathan and Ma (2003) that it is very difficult to forecast expected returns, one response to this is to set all expected returns to be equal to the same constant (for discussion of this, see Footnote 3). Then the mean-variance optimization problem in (1) reduces to the *minimum*-variance optimization problem, whose solution is,

$$w = \frac{\hat{\Sigma}^{-1}e}{e^{\top}\hat{\Sigma}^{-1}e} = \frac{(\mathrm{diag}(\hat{\sigma})\ \hat{\Omega}\ \ \mathrm{diag}(\hat{\sigma}))^{-1}e}{e^{\top}(\mathrm{diag}(\hat{\sigma})\ \hat{\Omega}\ \ \mathrm{diag}(\hat{\sigma}))^{-1}e}, \tag{5}$$

in which only two quantities need to be estimated: volatilities and correlations.

3 Data

In this section, we describe the data on stocks and stock options that we use in our study. Our data on stocks are from the Center for Research in Security Prices (CRSP) and NYSE's Trades-And-Quotes (TAQ) database. Our data for options is from IvyDB (OptionMetrics).

3.1 Data on Stock Returns

Our sample period is January 3, 1995 to June 29, 2007. We study stocks that are in the S&P500 index at any time during our sample period. The *daily* stock returns of the S&P500 constituents is from the daily file of the CRSP and we have in our sample a total of 3146 trading days. We also use high-frequency *intraday* stock-price data consisting of transaction prices of the S&P500 constituents; these data are from the NYSE's Trades-And-Quotes database. We use the intraday data because several studies have highlighted the advantage of using high-frequency data to measure volatility of financial returns, and also as a robustness check for the results obtained from daily data.[6]

To improve the quality of the raw data used in our analysis, we apply the following filters and data-cleaning rules. For the daily stock returns of the S&P500 constituents from the CRSP daily file, we remove the observations with standard missing codes (SAS missing codes A,B,C,D and E) as described in the Wharton Research Data Services documentation on CRSP. For the intraday stock-price raw data, we filter data for each day from the official opening at 9:30 EST until 16:00 EST, delete entries with a bid, ask or transaction price equal to zero, delete entries with corrected trades (trades with a correction indicator, "corr" $\neq 0$), delete entries with an abnormal sale condition (trades where the variable "cond" has a letter code, except for "E" and "F")[7] and delete entries with prices that are above the ask plus the bid-ask spread or prices that are below the bid minus the bid-ask spread; see Barndorff-Nielsen, Hansen, Lunde, and Shephard (2009) for the details and discussion of these rules.[8] After cleaning the data, we construct a regularly spaced one-minute price grid for every trading day using the volume-weighted average of all transactions within a given minute. If there is no price for a given minute, we fill it in with the previous available price.

Counting by IvyDB (OptionMetrics) identifiers, we have data for a maximum of 810 stocks, from which we choose those stocks for which at least 2,000 records of intraday volatilities and model-free implied volatilities are available, which gives us 561 stocks. Of these 561 stocks, there are 219 stocks for which the intraday volatilities and model-free implied volatilities are available for the *entire* time series. For robustness, we consider two datasets in our analysis. The first consists of the entire 561 stocks,[9] and the second consists of 100 stocks out of the 219 for which data are available for all dates; to select these 100 stocks, we first order the 219 stocks with respect to the security identifier code of the IvyDB data base, and then select the first 100.[10]

[6] For a survey of the literature on using high-frequency data to estimate moments of asset returns, see Andersen, Bollerslev, and Diebold (2009).

[7] See the TAQ 3 Users Guide for additional details about sale conditions.

[8] Rules P1, P2, T1, T2 and an adjusted version of T4.

[9] At each point in time, we consider only those stocks that have no missing data, which means that this sample has a variable number of stocks; on average, there are about 400 stocks at each point in time.

[10] In addition to the reported results, we have also checked our results on different subsamples of 50 and 100 stocks out of the 219 for which data are available for all dates, and these subsamples deliver similar results.

3.2 Data on Stock Options

For stock options we use the IvyDB that contains data on all U.S.-listed index and equity options. We use data from January 4, 1996 to June 29, 2007.[11]

We do not use option prices directly in our analysis, but wish to use option-based information only to obtain the moments of the option-implied distributions, and for this reason it is important for us to have the maximum number of options for a given maturity. Therefore, we choose for our analysis not the raw data on prices of options, but the volatility surface file, which contains a smoothed implied-volatility surface for a range of standard maturities and a set of option delta points.[12]

From the surface file we select for our sample the out-of-the-money implied volatilities for calls and puts (we take implied volatilities for calls with deltas smaller or equal to 0.5, and implied volatilities for puts with deltas bigger than -0.5) for standard maturities of 30 and 60 days, which we consider to be the most suitable.[13] For each date, each underlying stock, and each time to maturity, we have from the surface data 13 implied volatilities, which are then used to calculate the moments of the risk-neutral distribution.[14]

When working with data on option prices and the volatility surface, for several calculations we need a proxy for the riskfree rate for the maturity of a particular option. For this, we use the certificate-of-deposit yields for maturities between one day and one year from the IvyDB and interpolate them linearly to get the appropriate yield.

4 Description of Performance Metrics Used

We evaluate performance of the various portfolio strategies using four criteria. These are the (i) out-of-sample portfolio volatility (standard deviation); (ii) out-of-sample portfolio Sharpe ratio; (iii) out-of-sample portfolio certainty-equivalent return; and (iv) portfolio turnover (trading volume). The reason for considering the certainty-equivalent return, in addition to the Sharpe ratio, is that the Sharpe ratio considers only the mean and volatility of returns, while the certainty-equivalent considers also the higher moments of returns.

[11]Note that our data for stocks start in 1995, but we need 750 data points to compute the covariance matrix, so our portfolio optimization starts only at the beginning of 1998.

[12]We calculated implied moments also from the raw data on option prices, and the results are similar.

[13]The use of out-of-the-money options is standard in this literature; see, for instance, Bakshi, Kapadia, and Madan (2003) and Carr and Wu (2009). The reason is that selecting options that are out of the money reduces the effect of the premium for early exercise for these American options.

[14]There are 13 implied volatilities given for standard delta points for each call and put. For puts, these 13 are $\{-0.80, -0.75, -0.70, -0.65, -0.60, -0.55, -0.50, -0.45, -0.40, -0.35, -0.30. -0.25, -0.20\}$, and for calls the delta points are the same, but positive. We select calls with a delta less than or equal to 0.5 and for puts greater than -0.5, which gives a total of 13 implied volatilities for out-of-the-money options—a mix of calls and puts.

We use the following "rolling-horizon" procedure for computing the portfolio weights and evaluating their performance. First, we choose a window over which to perform the estimation. We denote the length of the estimation window by $\tau < T$, where T is the total number of returns in the dataset. For our experiments, we use an estimation window of $\tau = 750$ data points, which for daily data corresponds to three years.[15] Two, using the return data over the estimation window τ, we compute the various portfolios we wish to compare. Three, we repeat this "rolling-window" procedure for the next day, by including the data for the next day and dropping the data for the earliest day. We continue doing this until the end of the dataset is reached. At the end of this process, we have generated $T - \tau$ portfolio-weight vectors for each strategy; that is, $w_t^{strategy}$ for $t = \tau, \ldots, T-1$ and for each strategy.

Following this "rolling horizon" methodology, holding the portfolio $w_t^{strategy}$ for one day (or for one month, when we consider a monthly holding period) gives the *out-of-sample* return at time $t+1$: that is, $r_{t+1}^{strategy} = w_t^{strategy} r_{t+1}$, where r_{t+1} denotes the returns from t to $t+1$. After collecting the time series of $T - \tau$ returns, $r_t^{strategy}$, the out-of-sample mean, volatility ($\widehat{\sigma}$), Sharpe ratio of returns (SR), and certainty-equivalent return (ce) are:

$$\widehat{\mu}^{strategy} = \frac{1}{T-\tau} \sum_{t=\tau}^{T-1} r_{t+1}^{strategy}, \tag{6}$$

$$\widehat{\sigma}^{strategy} = \left(\frac{1}{T-\tau-1} \sum_{t=\tau}^{T-1} \left(r_{t+1}^{strategy} - \widehat{\mu}^{strategy} \right)^2 \right)^{1/2}, \tag{7}$$

$$\widehat{\mathrm{SR}}^{strategy} = \frac{\widehat{\mu}^{strategy}}{\widehat{\sigma}^{strategy}}, \tag{8}$$

$$\widehat{\mathrm{ce}}^{strategy} = u^{-1} \left(\frac{1}{T-\tau} \sum_{t=\tau}^{T-1} u\left(r_{t+1}^{strategy} \right) \right), \tag{9}$$

where u denotes the power utility function with a relative risk aversion of $\gamma = 1$, and the certainty-equivalent return (ce) is the riskless return that an investor is willing to accept instead of investing in the risky strategy.

To measure the statistical significance of the difference in the volatility, Sharpe ratio and certainty-equivalent return of a particular strategy from that of another strategy that serves as benchmark, we report also the p-values for the volatilities, Sharpe ratios and certainty-equivalent returns. We are interested in the finite-sample properties of the strategies. Therefore, when calculating the p-values for the case of daily rebalancing we use the bootstrapping methodology described in Efron and Tibshirani (1993), and for monthly rebalancing we make

[15] Because our sample consists of 561 stocks, shorter estimation window lengths such as τ=500 and 250 often give singularities in the covariance matrix.

an additional adjustment, as in Politis and Romano (1994), to account for the autocorrelation arising from overlapping returns.[16]

Finally, we wish to obtain a measure of portfolio turnover. Let $w_{j,t}^{strategy}$ denote the portfolio weight in stock j chosen at time t under strategy *strategy*, $w_{j,t^+}^{strategy}$ the portfolio weight *before* rebalancing but at $t+1$, and $w_{j,t+1}^{strategy}$ the desired portfolio weight at time $t+1$ (after rebalancing). Then, turnover, which is the average percentage of wealth traded per rebalancing interval (daily or monthly), is defined as the sum of the absolute value of the rebalancing trades across the N available stocks and over the $T-\tau-1$ trading dates, normalized by the total number of trading dates:

$$\text{Turnover} = \frac{1}{T-\tau-1} \sum_{t=\tau}^{T-1} \sum_{j=1}^{N} \left(\left| w_{j,t+1}^{strategy} - w_{j,t^+}^{strategy} \right| \right). \tag{10}$$

5 Benchmark Strategies that Do Not Use Option Prices

For robustness, we consider four benchmark strategies that are *not* based on option-implied information. These are: (i) the equally-weighted ($1/N$) strategy; (ii) the unconstrained minimum-variance strategy; (iii) the short-sale-constrained minimum-variance strategy; and (iv) the unconstrained minimum-variance strategy with shrinkage of the covariance matrix. The construction of these four benchmark strategies is described below. In principle, one could also consider the mean-variance portfolio as a benchmark but, as we discuss below, this performs much worse than our four benchmarks listed above.

For the "*equally-weighted*" ($1/N$) strategy, one invests an equal amount of wealth across all N available stocks each period. The reason for considering this strategy is that DeMiguel, Garlappi, and Uppal (2009) show that it performs quite well even though it does not rely on any optimization; for example, the Sharpe ratio of the S&P500 stocks over our sample period is 0.35 while that of the $1/N$ strategy is more than 0.90. We consider two rebalancing intervals: daily and monthly. For the monthly rebalancing interval, we find the new set of weights daily,

[16]Specifically, consider two portfolios i and n, with μ_i, μ_n, σ_i, σ_n as their true means and volatilities. We wish to test the hypothesis that the Sharpe ratio (or certainty-equivalent return) of portfolio i is worse (smaller) than that of the benchmark portfolio n, that is, $\text{H}_0 : \mu_i/\sigma_i - \mu_n/\sigma_n \leqslant 0$. To do this, we obtain B pairs of size $T-\tau$ of the portfolio returns i and n by simple resampling with replacement for daily returns, and by blockwise resampling with replacement for overlapping monthly returns. We choose $B = 10,000$ for both cases and the block size equal to the number of overlaps in a series, that is, 20. If $\hat{F}$ denotes the empirical distribution function of the B bootstrap pairs corresponding to $\widehat{\mu}_i/\widehat{\sigma}_i - \widehat{\mu}_n/\widehat{\sigma}_n$, then a one-sided P-value for the previous null hypothesis is given by $\hat{p} = \hat{F}(0)$, and we will reject it for a small $\hat{p}$. In a similar way, to test the hypothesis that the variance of the portfolio i is greater (worse) than the variance of the benchmark portfolio n, $\text{H}_0 : \sigma_i^2/\sigma_n^2 \geqslant 1$, if $\hat{F}$ denotes the empirical distribution function of the B bootstrap pairs corresponding to: $\hat{\sigma}_i^2/\hat{\sigma}_n^2$, then, a one-sided P-value for this null hypothesis is given by $\hat{p} = 1 - \hat{F}(1)$, and we will reject the null for a small $\hat{p}$. For a nice discussion of the application of other bootstrapping methods to tests of differences in portfolio performance, see Ledoit and Wolf (2008).

but hold that portfolio for 30 calendar days (21 trading days); therefore, this corresponds to the average of 21 daily returns. The advantage of this approach is that it is not sensitive to the particular day on which the portfolio is formed.

The second benchmark strategy that we consider is the minimum-variance portfolio using the "*sample covariance*" matrix. To identify the portfolio weights for this strategy, we need to estimate only the covariance matrix, that is, the realized volatilities and correlations. For *daily data* we compute the conventional sample estimators of (co)variance using data over the past 750 days.[17] In the existing literature, two methods have been adopted to improve the out-of-sample performance of the minimum-variance portfolio based on the sample (co)variances. One approach is to impose constraints on the portfolio weights, which Jagannathan and Ma (2003) show can lead to substantial gains in performance. Thus, our third benchmark is the "*constrained*" strategy, where we compute the short-sale-constrained minimum-variance portfolio weights. That is, we use the same approach as described above to compute the minimum-variance portfolio for daily and intraday data, but impose a non-negativity constraint when minimizing the variance of the portfolio.

Another approach to improve the out-of-sample performance of the minimum-variance portfolio based on the sample covariance matrix is to use "shrinkage." Our fourth benchmark consists of "*shrinkage*" portfolios, where we compute the minimum-variance portfolio weights *after* shrinking the covariance matrix.[18] The sample covariance matrix for daily data and intraday data is computed using the same approach that is described above. To shrink the covariance matrix for daily returns, we then use the approach in Ledoit and Wolf (2004a,b), where they show how one can compute the *optimal* shrinkage of the covariance matrix under certain assumptions about the distribution of returns.[19]

In **Table 1** we report the performance of these four benchmark strategies, all of which do *not* use data on option prices. In Panel A, we report the results for daily rebalancing, and in Panel B, we report the results when the portfolio is held for a month. Two p-values are reported in parenthesis under each performance metric. The first p-value is relative to the $1/N$ benchmark and the second p-value in this table is relative to the "Sample-cov" benchmark. Each p-value is for the *one-sided* null hypothesis that the policy being evaluated is *worse* than

[17]For the realized (co)variance estimators based on *intraday data* we use the filtered and calendar-time aligned transaction prices over the last 30 trading days to estimate the (co)variances. There are several issues that have to be addressed when estimating moments from intraday data; the approach we use is consistent with the "second-best" approach of Zhang, Mykland, and Aït-Sahalia (2005), and the details of our procedure are provided in Appendix A.

[18]We do not consider the norm-constrained approach of DeMiguel, Garlappi, Nogales, and Uppal (2009) because we already consider the shortsale-constrained and shrinkage portfolios, which are particular cases of the norm-constrained portfolios.

[19]For intraday data, instead of shrinkage, we use the regularization approach of Zumbach (2009) because the distribution of intraday returns is different from that of daily returns and does not satisfy the assumptions of Ledoit and Wolf (2004a,b). Details of the shrinkage and regularization methods we use are provided in Appendix B, and the results for intraday data are summarized in Section 8.2.

the benchmark for a given performance metric (so a small p-value suggests *rejecting* the null hypothesis that the policy being evaluated is worse than the benchmark).

From Table 1, we see that compared to the $1/N$ strategy most of the strategies based on the minimum-variance portfolio achieve significantly lower volatility ($\widehat{\sigma}$) out-of-sample. For example, in Panel A with results for "Daily rebalancing", we see that for the data with 561 stocks, the volatility of the $1/N$ portfolio is 0.1745 and that of the minimum-variance portfolio with daily data is 0.1347, for the minimum-variance portfolio with constraints is 0.1240, and for the minimum-variance portfolio with shrinkage is 0.1180. The first set of p-values indicate that the volatilities of the three minimum-variance policies are significantly lower than that of $1/N$; the second set of p-values indicate that the constrained and shrinkage policies have a lower portfolio volatility than the policy based on the sample covariance. The results for the data with 100 assets and in Panel B for "Monthly rebalancing" are similar.

However, the Sharpe Ratio (sr) and also the certainty-equivalent return (ce) is typically higher for the $1/N$ strategy. The one-sided p-values again indicate that we cannot reject the null that the Sharpe ratio of the minimum-variance policies is worse than that of $1/N$. These results are similar for "Monthly rebalancing" reported in Panel B for the data with 100 stocks, except that for the data with 561 stocks the Sharpe ratio and certainty-equivalent return are highest for the constrained strategy rather than $1/N$.

Of the three minimum-variance strategies, the short-sale-constrained strategy has the lowest turnover (which is true also in the tables that follow where we use option-implied information). However, the turnover of the $1/N$ strategy is almost always lower than even that of the constrained minimum-variance portfolio strategy.

For completeness, we also discuss briefly the results for the *mean-variance* portfolio strategies. Of the three variants of the mean-variance portfolio we consider, the first is based on the sample covariance matrix, the second has short-sale constraints, and the third is computed with shrinkage applied to the covariance matrix, as in Ledoit and Wolf (2004a,b). All three mean-variance portfolio policies perform very poorly along all metrics. For example, while the volatility of the three minimum-variance portfolios is less than 0.1350, the volatility of the corresponding three mean-variance policies is at least 3 times higher. Similarly, the Sharpe ratio of the short-sale-constrained mean-variance policies is less than half of that of the short-sale-constrained minimum-variance portfolios, and it is negative for the other two mean-variance policies. Finally, the turnover of the short-sale-constrained mean-variance strategy is five times that of the minimum-variance strategy, and that for the other two mean-variance strategies is about fifteen times higher. Consistent with the findings documented in the existing literature, we conclude that relative to the $1/N$ portfolio and also the minimum-variance benchmarks, the mean-variance portfolios perform much worse across all four metrics.

6 Option-Implied Information

In this section, we explain how we compute the quantities implied by option prices that we use in our selection of portfolios. These quantities are: (i) option-implied volatility and the volatility risk premium; (ii) option-implied correlation; and, (iii) option-implied skewness. We also examine the ability of option-implied volatility and option-implied correlation to predict realized volatility and realized correlation, respectively.

6.1 Option-Implied Volatility and Volatility Risk Premium

When option prices are available, an intuitive first step is to use this information to back out implied volatilities. In contrast to the model-specific Black-Scholes implied volatility, we use the *model-free implied volatility* (MFIV), which represents a nonparametric estimate of the risk-neutral expected stock-return volatility until the option's expiration.

Model-free implied volatility is given by a single number and it subsumes information in the whole Black-Scholes implied volatility smile. Theoretical and empirical research (see Jiang and Tian (2005) and Vanden (2008)) finds that model-free implied volatility is better at predicting the future realized volatility than the Black-Scholes implied volatility, and it is used by the CBOE to compute VIX, which is the ticker symbol for the CBOE Volatility Index that gives the implied volatility of S&P500 index options. To compute the model-free implied volatility, we first calculate the option prices from the interpolated volatility surface data. We then use these prices to find the value of the "variance contract," following the approach in Bakshi, Kapadia, and Madan (2003); the formula for the variance contract and the procedure used to compute it is provided in Appendix C.[20] The square root of the variance contract then gives us the model-free implied volatility.

To verify whether the intuition that the model-free implied volatility is a useful predictor of realized volatility in the future, we regress realized variances on the model-free implied variances and compare the R^2 with that when variances based on historical data are used as a predictor. We see from Panel A of **Table 2** that when regressing the 30-day realized variances in the future on (i) 750-day historical daily variances, (ii) 30-day intraday historical variances, and (iii) model-free implied variances, the R^2 for the model-free implied variances is higher than

[20]For a discussion of how to compute the model-free implied volatility, see also Dumas (1995), Carr and Madan (1998, 2001), and Britten-Jones and Neuberger (2000).

that for intraday historical variances, which is higher than for daily historical variances. This is true for both the dataset with 100 stocks and that with 561 stocks.[21]

However, what we need for portfolio selection is not the risk-neutral implied volatilities of stock returns but the expected volatilities under the objective distribution. We now explain how to make a correction to the model-free implied volatilities in order to get the volatilities under the objective measure.

The difference between the model-free implied volatility and the expected volatility is the *volatility risk premium*. Bollerslev, Gibson, and Zhou (2004), Carr and Wu (2009), and others have shown that one can use the realized volatility (instead of the expected volatility) to estimate the volatility risk premium. Under the assumption that the magnitude of the variance risk premium is proportional to the level of the variance under the actual probability measure (as it is in the Heston (1993) model), we estimate the historical volatility risk premium (HVRP) as the square root of the average variance risk premium for each stock on each day for the past period of T trading days:[22]

$$\text{HVRP}_t^2 = \frac{1}{T - \Delta t} \sum_{i=t-T+1}^{t-\Delta t} \frac{\text{MFIV}_{i,i+\Delta t}^2}{\text{RV}_{i,i+\Delta t}^2}. \tag{11}$$

In our analysis, we estimate the historical volatility risk premium on each day over the past year (-252 days to -21 days) using the model-free implied volatility and realized volatility, each measured over 21 trading days and each annualized appropriately. Then, assuming that in the next period, from t to $t + \Delta t$, the prevailing volatility risk premium will be well approximated by the historical volatility risk premium in (11), one can obtain the *prediction* of the future realized volatility, $\widehat{\text{RV}}_t$:

$$\widehat{\text{RV}}_{t,t+\Delta t} = \frac{\text{MFIV}_{t,t+\Delta t}}{\text{HVRP}_t}. \tag{12}$$

Panel A of Table 2 shows that for the data with 561 stocks the R^2 for the regression of the risk-premium-corrected implied volatility is equal to 31.55%, which is about the same as the R^2 for the model-free implied volatility, suggesting that there is not much improvement in the prediction ability.

A visual representation of the prediction power of implied volatility is given in **Figure 1**, where we plot the historical volatility based on the last 750 days (dot-dashed blue line), model-free implied volatility (dashed red line), risk-premium-corrected model-free implied volatility

[21] The results are similar also when we regress the 30-day intraday (high-frequency) volatilities on the same regressors.

[22] Note that because HVRP_t^2 is calculated as the average of the ratio of $\text{MFIV}_{i,i+\Delta t}^2$ and $\text{RV}_{i,i+\Delta t}^2$, both of which are calculated over Δt days, as a result we will have only $T - \Delta t$ observations when computing the average.

(solid pink line), and the 30-day realized volatility (thick black line). The figure is based on the cross-sectional equally-weighted average volatilities across the 561 stocks at each point in time. The figure shows that the risk-premium-corrected model-free implied volatility tracks realized volatility quite closely. The model-free implied volatility (without any risk-premium correction) tracks the realized volatility, but there is a distinct gap between the two. And, the historical 750-day realized volatility does not track realized volatility very closely. Note also each of these volatility series has a different level of variability.

6.2 Option-Implied Correlation

The second piece of option-implied information that we consider is implied correlation. It has been shown by Driessen, Maenhout, and Vilkov (2009) that the risk-neutral expectation of the correlation is higher than the expectation under the objective probability measure. Therefore, one approach for computing implied correlation is to take the historical correlation matrix as the proxy for the future realized correlation matrix and increase each pairwise historical correlation by the correlation risk premium. We use the same methodology as in Buss and Vilkov (2008) to obtain implied correlation. That is, we assume that the size of the adjustment to each pairwise historical correlation is proportional to the distance between the calculated historical correlation and the perfect correlation of one, which ensures that the implied correlation matrix satisfies the necessary positive-definiteness restriction.

Having computed implied correlations using the above approach, we examine whether they are superior at predicting realized correlations. To do this, we regress realized correlations on (i) 750-day historical daily correlations, (ii) 750-day historical daily correlations after shrinkage, (iii) 30-day intraday correlations, (iv) 30-day intraday correlations after regularization, and (v) implied correlations, as computed in Buss and Vilkov (2008). We report in Panel B of **Table 2** that for the data with 561 stocks, the R^2 for historical daily correlations is 3.97%, for historical daily correlations with shrinkage is 3.28%, for intraday correlations is 6.26%, for intraday correlations with shrinkage is 4.79%, and for implied correlation is 9.32%, with the results being similar for the data with 100 stocks. Thus, while high-frequency intraday correlations do not seem to predict realized correlations better than historical correlations based on daily data, implied correlations are better, though the improvement in R^2 is smaller than it was for predicting realized volatilities.

In **Figure 2**, we plot the historical correlation based on the last 750 days (dashed blue line), implied correlation (solid red line), and 30-day realized correlations (thick black line). Just as the figure for volatilities, the plot is based on the cross-sectional equally-weighted average of average correlations across 561 stocks. There are two observations about these series: first, implied correlation follows the level of realized correlation much more closely than historical correlation; two, implied correlation is much more volatile (that is, contains more noise) than realized correlation, while historical correlation is almost a smooth function (but contains much less of the current information).

6.3 Option-Implied Skewness

The *model-free implied skewness* represents a nonparametric estimate of the risk-neutral stock-return skewness, and it is this skewness that gives rise to the Black-Scholes implied volatility smile. Some researchers use as a simple measure of skewness the difference between the implied-volatilities for out-of-the-money and at-the-money put options (Xing, Zhang, and Zhao, 2009), however that measure does not take into account the whole distribution, but rather just the left tale. Our calculation of the model-free implied skewness (MFIS) parallels that of the model-free implied volatility. We first calculate the option prices from the interpolated volatility surface data. We then use these prices to determine model-free implied skewness as described by Bakshi, Kapadia, and Madan (2003); the formula for this and the procedure used to compute it is provided in Appendix C.

7 Portfolios that Use Option-Implied Information

In this section, we discuss the major findings of our paper about the ability of forward-looking information in option prices to improve the out-of-sample performance of stock portfolios. As explained above, to determine mean-variance portfolio weights, one needs to estimate for each stock its (i) volatility, (ii) correlations with all other stocks, and (iii) expected return. Accordingly, we divide our analysis into three parts. In Section 7.1, we examine how option-implied volatilities can be used for portfolio selection. In Section 7.2, we study how option-implied correlations can be used for portfolio selection. Finally, in Section 7.3, we investigate the performance of portfolios based on expected returns adjusted by option-implied information. In each of these sections, we obtain only one of the three moments using option-implied information in order to identify the magnitude of the gains from that particular source of information.

7.1 Portfolios Using Option-Implied Volatilities

Motivated by the findings in Section 6.1 about the predictive power of model-free implied volatilities, we use them in $\text{diag}(\hat{\sigma})$ to obtain the covariance matrix given in (4); that is, we use as the covariance matrix: $\hat{\Sigma} = \text{diag}(\text{MFIV})\, \hat{\Omega}\, \text{diag}(\text{MFIV})$. Using this adjusted covariance matrix, we then determine the optimal portfolio strategies in (5), along with the strategies where short sales are constrained, and where the shrinkage is applied to this covariance matrix. Note that in computing these optimal strategies, we continue to use historical correlations, and we set expected returns on all stocks to be equal.[23] Effectively, we are computing the minimum-variance portfolio but with implied volatilities instead of historical volatilities in the covariance matrix. The results are reported in **Table 3**. This table and also all the remaining tables have two sets of p-values: the first with respect to the $1/N$ strategy, and the second with respect to the corresponding minimum-variance benchmark policy in Table 1.

Table 3 shows that the optimal portfolio strategies based on the model-free implied volatility have a lower volatility than $1/N$, but the Sharpe ratio, certainty-equivalent return, and turnover is better for $1/N$. Compared to the traditional minimum-variance portfolios in Table 1 that are based on historical data, we find that replacing the estimate of stock-return volatilities by their model-free implied counterparts always helps to reduce the out-of-sample volatility of the portfolio, although the reduction itself is relatively small (by an average of 1.6% for daily rebalancing and 4.7% for monthly rebalancing) and the p-values for this reduction are significant only for the data with 100 stocks for monthly rebalancing.

There are two striking conclusions from Table 3, both of which are *negative*. First, the reduction in portfolio volatility is very small. Second, and more damaging, is the observation that the Sharpe ratios and also the certainty-equivalent returns are substantially *smaller* for the strategies in Table 3 that are based on implied-volatility, compared to those in Table 1, which rely only on historical estimates of volatility. One reason for these negative results is that when we derive the model-free implied volatility from options, we are getting the *risk-neutral* volatility, which is the sum of the volatility risk premium and expected volatility under the objective measure; thus, implied volatility is a biased estimator of expected volatilities under the objective distribution. Moreover, assuming the same level of expected volatility under the objective measure, the implied volatility is relatively higher for stocks with high volatility risk premium than for stocks with low volatility risk premium. Hence, when we use risk-neutral implied volatilities, we underweight the stocks with high volatility risk premium (because they have a higher implied volatility) in comparison to the stocks with low volatility risk premium. Given the findings of Goyal and Saretto (2009) that stocks with high volatility risk premium have higher returns, this explains the reduction in the portfolio's realized return, and hence, its Sharpe ratio.

Table 4 gives the results where volatility of stock returns is estimated from the model-free implied volatility *after* it is corrected for the risk premium. Comparing the results in this table to those in Table 3, where implied volatility is not corrected for the risk premium, we see that

[23] As explained above, the reason for setting expected returns to be equal for all stocks in the first two steps is that using historical estimates of expected returns leads to portfolios with very poor performance, as discussed at the end of Section 5 and also in Jagannathan and Ma (2003). See also Footnote 3.

using the risk-premium-corrected implied volatilities leads to a reduction in portfolio volatility, especially for the case of 561 assets. Comparing the results in Table 4 to the traditional minimum-variance portfolios in Table 1, we observe that the portfolios with the risk-premium-corrected implied volatilities attain a lower out-of-sample volatility, with the difference being around 6%–10% for monthly rebalancing and around 0–7% for daily rebalancing.

More importantly, comparing Table 4 with Table 3, we see that there is a significant improvement in the Sharpe ratios and certainty-equivalent returns and turnovers when using the risk-premium-corrected implied volatilities, which confirms our motivation for correcting the implied-volatilities for the volatility risk premium. However, the Sharpe ratios and certainty-equivalent returns in Table 4, are still lower than those for the sample-based benchmark portfolios in Table 1. And, for almost all the cases considered in Table 4, the $1/N$ strategy has a higher Sharpe ratio and certainty-equivalent return, and substantially lower turnover (the exception is the shortsale-constrained strategy for 561 stocks with monthly rebalancing, though the p-value is not significant). The reason for the relatively poor Sharpe ratio and certainty equivalent return of the strategies based on implied volatility is that, with or without the risk-premium correction, implied volatility is highly variable over time (see Figure 1), which increases the instability of portfolio weights and reduces the gains from having a better predictor of realized volatility.

7.2 Portfolios Using Option-Implied Correlations

Next, we investigate the use of option-implied correlations in portfolio selection. In order to isolate the effect of using implied correlations, when computing portfolios we use volatilities from historical data, and we continue to set expected returns across all assets to be equal.

Table 5 shows that of the three minimum-variance strategies we consider with the implied correlations, the constrained strategy does best; but the portfolios relying on implied correlations typically perform worse than the corresponding minimum-variance benchmark portfolios in Table 1 in terms of all four performance metrics: volatility, Sharpe ratio, certainty-equivalent return, and turnover. One possible explanation for this poor performance is that by replacing historical correlations by the implied correlations, we are essentially increasing the magnitude of the off-diagonal elements of the covariance matrix, making the covariance matrix less stable, which then leads to poor performance. Moreover, as can be seen in Figure 2, the implied correlations are also much more variable than the other series; this variability is reflected also in Panel B of Table 2 where the p-values of the β estimated in the predictive regressions is very low, even though the R^2 is the highest when implied correlation is the predictor.[24]

[24]In order to conserve space, we do not present the table where the portfolios are computed using implied correlations *and* risk-premium-corrected implied volatilities. The table with the results for this case is available

7.3 Portfolios Using Expected Returns Adjusted by Option-Implied Information

In the two subsections above, we set expected returns to be equal across all stocks, while examining the effect of using option-implied volatilities and correlations. The reason for setting expected returns to be equal is that (i) portfolio weights are very sensitive to expected returns, and (ii) estimates of expected returns from historical data are extremely noisy, making it difficult to distinguish whether expected returns are really different across stocks.[25] Consequently, as shown in Table 1, portfolios based on expected returns estimated from historical data perform very poorly.

In this subsection, we examine if it is possible to use option-implied information to adjust the naive procedure of setting expected returns to be equal across all stocks. In Section 7.3.1, we examine the effect of adjusting expected returns based on the volatility risk premium, and in Section 7.3.2, we adjust expected returns based on option-implied skewness.

7.3.1 Expected returns scaled by historical volatility risk premium

Goyal and Saretto (2009) have documented that assets with high volatility risk premium tend to outperform those with low volatility risk premium.[26] We now study whether this empirical regularity can be exploited to improve portfolio performance.

To incorporate the above finding for portfolio selection, we start by setting all expected returns to be equal to one. We then sort all the stocks into deciles using the characteristic "historical volatility risk premium," and then scale the expected returns of the top decile to $(1 + \delta)$ and of the bottom decile to $(1 - \delta)$. In our empirical implementation, to give the reader a sense of how the results change with δ, we consider two values of the scaling factor: $\delta_1 = 0.50$ and $\delta_2 = 0.70$. We then use the scaled expected returns, along with volatilities and correlations estimated from historical data, to compute the optimal portfolios as in (3). The performance of the portfolio obtained in this fashion is given in **Table 6**.

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from the authors. As one would expect, the results are a combination of the insights from Tables 4 and 5. What one can conclude from these results is that using implied correlations and risk-premium-corrected implied volatilities has (i) a mixed effect on portfolio volatility, reducing it in some cases but increasing it in others; (ii) reduces Sharpe ratios and certainty-equivalent returns considerably; and (iii) usually increases turnover. Compared to $1/N$, the strategy using option-implied volatilities *and* correlations is better in terms of portfolio volatility, but worse in terms of Sharpe ratio, certainty-equivalent return, and turnover.

[25] Simulations in DeMiguel, Garlappi, and Uppal (2009) suggest that more than five hundred years of data are needed before one can estimate expected returns with sufficient precision to improve the performance of a portfolio with fifty risky assets beyond that of the $1/N$ portfolio.

[26] See also Cremers and Weinbaum (2008) for another characteristic of options that is useful for predicting stock returns. They find that deviations from put-call parity contain information about future stock returns. They measure these deviations from the difference in implied volatility between pairs of call and put options and find that stocks with relatively expensive calls outperform stocks with relatively expensive puts.

We see from Table 6 that the portfolios with expected returns scaled by the volatility risk premium have a volatility that is typically lower than that of the $1/N$ portfolio, but higher than that of the benchmark portfolios in Table 1, with a higher δ leading to an increase in the portfolio volatility. However, now the Sharpe ratios and certainty-equivalent returns for the strategies whose expected returns have been scaled by the volatility risk premium exceed those of $1/N$ for the data with 100 assets for all cases, and for the data with 561 assets for the case where short sales are constrained (but with p-values that are not low). However, the improvement in Sharpe ratios is accompanied by an increase in turnover. And, increasing δ increases the Sharpe ratio but also turnover.[27]

One limitation of scaling expected returns is that the portfolio weights are very sensitive to even small errors in estimates of expected returns, which then lead to high turnover. In order to incorporate the effect of the historical volatility risk premium on portfolio weights without a significant increase in turnover, we consider scaling stock-return volatilities instead of expected returns. That is, we scale the sample estimate of $\hat{\sigma}$ that we are using in the covariance decomposition (4). To make this adjustment, we proceed as before: we first sort stocks into deciles using the characteristic "historical volatility risk premium," and then scale the volatilities of the top decile to $\hat{\sigma} \cdot (1 - \delta)$ and of the bottom decile to $\hat{\sigma} \cdot (1 + \delta)$, using the same scaling factors as before: $\delta_1 = 0.50$ and $\delta_2 = 0.70$.

Table 7 gives the results when the estimates of the historical volatilities have been scaled, based on the volatility risk premium for each stock. The table shows that scaling the stock return volatilities improves performance across all four metrics compared to scaling expected returns, as in Table 6. Moreover, the Sharpe ratio and certainty equivalent return are typically greater than for the benchmark minimum-variance portfolios in Table 1 (the only exception are the constrained strategies for 561 stocks). Moreover, in all cases but that of the sample covariance portfolio for 561 stocks, the Sharpe ratios exceed that of the $1/N$ strategy (with several p-values being below 0.10). And, the increases in Sharpe ratios are, interestingly, accompanied by a *decrease* in the portfolio turnover, which are now lower than those in Table 6. Likewise, the table shows that the scaling based on the volatility risk premium always helps to increase the out-of-sample certainty-equivalent of portfolio returns, and although the increase is not as large as for the Sharpe ratio, it ranges from 1.5% and 6.4% for daily rebalancing and 1.3% to 3.9% for monthly rebalancing. We also see that increasing δ *reduces* turnover, increases portfolio volatility, and has a mixed effect on Sharpe ratios and certainty-equivalent returns.

[27] Again, we do not report the results where expected returns are scaled based on the volatility risk premium *and* one uses the risk-premium-corrected implied volatilities. These results are available from the authors. Not surprisingly, compared to Table 6 portfolio volatility is now lower; Sharpe ratios and certainty-equivalent returns improve for the unconstrained strategies, but get worse for the short sale constrained strategy and exceed those for the $1/N$ strategy for the data set with 100 stocks but not for the data set with 561 stocks; and, turnover is lower for the unconstrained strategies but higher for the short sale constrained strategy.

We conclude from the comparison of the results in Table 7 and Table 1 that the volatility risk premium is useful for improving portfolio performance; and from the comparison of Table 7 and Table 6, that scaling volatilities is better than scaling the expected returns.

7.3.2 Expected returns scaled by implied skewness

The final approach we consider for identifying portfolios with high out-of-sample performance is motivated by the empirical results of Rehman and Vilkov (2008), who find that stocks with high option-implied skewness outperform stocks with low option-implied skewness. To exploit this feature of the data, we proceed in the same manner as in the section above.

We start by setting all expected returns to be equal to one. We then sort all the stocks in the data set by the characteristic "model-free implied skewness" into deciles, and then scale the expected returns of the top decile to $(1+\delta)$ and of the bottom decile to $(1-\delta)$. We consider two values of the scaling factor: $\delta_1 = 0.30$ and $\delta_2 = 0.50$.[28] We then use the scaled expected returns, along with volatilities and correlations estimated from historical data, to compute the optimal portfolios as in (3). The results for the portfolio weights obtained in this fashion are given in **Table 8**.

We see from Table 8 that the portfolios with expected returns scaled by implied skewness perform very favorably compared to $1/N$: the portfolios with scaled expected returns usually have lower volatility for the case of $\delta_1 = 0.30$, along with higher Sharpe ratios and certainty-equivalent returns (with these differences being statistically significant for daily rebalancing in Panel A for both the data with 100 stocks and that with 561 stocks). However, this is accompanied by substantially higher turnover. Compared to the other benchmark portfolios in Table 1, the differences in Sharpe ratios and certainty-equivalent returns are even greater, though the benchmark portfolios have lower portfolio volatility and turnover. Increasing δ increases the Sharpe ratio and certainty equivalent return, but also increases turnover and portfolio volatility.

In an effort to reduce portfolio turnover, we again consider scaling volatilities instead of expected stock returns. To scale the volatilities in the covariance decomposition (4), we first sort stocks into deciles using the characteristic "model-free implied skewness" and then scale the volatilities of the top decile to $\hat{\sigma} \cdot (1-\delta)$ and for the bottom decile to $\hat{\sigma} \cdot (1+\delta)$. We consider the same two values for the scaling factor: $\delta_1 = 0.30$ and $\delta_2 = 0.50$.

Comparing **Table 9** to Table 8 shows that scaling the stock-return volatilities rather than expected returns reduces turnover without reducing performance in terms of volatility, Sharp ra-

[28] Note that we use smaller values of δ than for the scaling based on the volatility risk premium in the previous section. This is because the model-free implied skewness has a stronger effect on the portfolio's out-of-sample performance.

tio, and certainty-equivalent return (except for the short-sale-constrained strategy). Comparing Table 9 to the benchmark portfolios in Table 1, we see that using the option-implied skewness leads to a significant increase in the out-of-sample Sharpe ratio, which ranges between 30% and 400% for daily rebalancing, and between 11% and 87% for monthly rebalancing. However, this is accompanied by a considerable increase in turnover.

Comparing Table 9 to Table 7, we see that the Sharpe ratios obtained from using implied skewness are higher than those from using the volatility risk premium. The main difference between the results obtained with the scaling based on the model-free implied skewness and that based on the historical volatility risk premium is the effect on portfolio turnover. While the adjustment based on implied skewness increases Sharpe ratio but also leads to a much higher turnover, the adjustment based on the volatility risk premium achieves improvement in Sharpe ratios while often *reducing* the portfolio turnover. The reason for this is that the historical volatility risk premium estimator has much lower estimation variance than the model-free implied skewness. This is because the model-free implied skewness is estimated purely from current option prices,[29] and therefore is based on the market's expectations about the future, while the historical volatility risk premium is computed from historical return data as well as historical model-free implied volatilities, and therefore, is more stable.

Overall, the empirical evidence demonstrates that using the volatility risk premium and model-free implied skewness can lead to a substantial improvement in the out-of-sample portfolio Sharpe ratios and certainty-equivalent returns, with the main challenge being how to control turnover.

8 Robustness Tests

In this section, we describe the various tests that we have undertaken to verify the robustness of the results from our empirical analysis.

8.1 Different Datasets

Ideally, one would like to study more than a single dataset. We are limited in our desire to consider additional datasets because we have option prices only for U.S. stocks. To overcome this limitation, we undertake our analysis for two datasets, where the second dataset is a subset of the first. The first consists of the entire 561 stocks in our dataset, and the results for this dataset are reported in the last four columns of each table. The second dataset consists of 100 stocks out of the 219 for which data are available for all dates, where these 100 stocks

[29] In order to reduce variability of implied skewness, we use its average value over the last five days but it is still quite variable.

are selected by first sorting the 219 stocks with respect to the security identifier code of the IvyDB data base, and then selecting the first 100. The results for this dataset are reported in Columns 2–4 of each table. In addition to the reported results, we have also checked our results on different subsamples of 50 and 100 stocks out of the 219 for which data are available for all dates, and these subsamples deliver similar results.

8.2 Different Data Frequencies

We consider both *daily* stock returns from the CRSP daily file and *intraday* stock-price data from the NYSE's Trades-And-Quotes (TAQ) database. Results for the daily data are reported under the heading of "Daily data" in each panel of each table. Results for the high-frequency transaction data are not reported in order to conserve space, and can be obtained from the authors. The main insight from intraday data is that using this high-frequency data to compute the covariance estimators rarely improves the out-of-sample performance of resulting portfolios, and in our analysis the intraday data give significantly better results relative to daily data only for the sample covariance matrix for 561 assets when neither shrinkage is applied to the covariance matrix nor short-sale constraints are imposed on the portfolio weights.

8.3 Different Rebalancing Frequencies

We consider two rebalancing frequencies in our analysis: *daily* rebalancing, the results for which are given in Panel A of each table, and *monthly* rebalancing, the results for which are given in Panel B of each table. We find that the results are in the same direction for the two holding periods, though Sharpe ratios and certainty equivalent returns are higher for daily rebalancing, while turnover is lower for monthly rebalancing.

8.4 Different Benchmark Strategies

We consider four benchmark strategies, all of which are listed in Table 1. The first is the equally-weighted ($1/N$) strategy in which one invests an equal amount of wealth across all N available stocks each period. As DeMiguel, Garlappi, and Uppal (2009) have shown, this is strategy that performs extremely well. For example, its Sharpe ratio over our sample period exceeds 0.90, while that for the S&P500 index over the corresponding period is only 0.35. The second benchmark strategy is the minimum-variance portfolio using daily data to compute the sample covariance matrix. The third benchmark strategy we consider is the minimum-variance portfolio with shortsale constraints, which is motivated by the finding in Jagannathan and Ma (2003) that imposing short-selling constraints can lead to substantial gains in performance. The fourth benchmark strategy we consider is the minimum-variance portfolio with "shrinkage", using the approach in Ledoit and Wolf (2004a,b) for daily data, and with regularization using the approach of Zumbach (2009) for intraday data.

Note that we do not consider the mean-variance portfolio as one of the benchmark strategies, because the performance of this portfolio is quite poor, as already documented extensively in the literature; see, for instance, DeMiguel, Garlappi, and Uppal (2009). For completeness, the performance of the mean-variance portfolio for daily returns is discussed at the end of Section 5.

8.5 Different Objective Functions

In our analysis, our objective has been either mean-variance optimization or minimum-variance optimization (when all expected returns are set equal to 1). However, one could have used a utility function instead. When we use the log utility function (that is, the power utility function with risk aversion γ equal to 1), we find for the data with 100 assets that the optimal portfolio has extreme weights and it performs extremely poorly, just as the mean-variance portfolio. In order to compare the weights with those from the optimization of the minimum-variance objective function, we "demean" the returns and set expected returns on all assets to be equal to 1. We then repeat the maximization of the log utility function and find that the weights are virtually identical to the minimum-variance weights.

When maximizing log utility, we also consider scaling expected returns based on the historical volatility risk premium and the option-implied skewness to scale expected returns (just as we did above, by adjusting the top and bottom deciles of assets); we find that the weights are too sensitive to these adjustments to expected returns. We then consider scaling the volatilities of stock returns by multiplying the returns with the same scaling factor, δ, that we used for the mean-variance analysis in Tables 7 and 9. We find a distinct improvement in portfolio performance, and the scaling works in the same direction as reported in these tables.

9 Conclusion

Mean-variance portfolios depend on estimates of volatilities, correlations, and expected returns of stocks. In this paper, we have studied how information implied in prices of stock options can be used to estimate these three moments in order to improve the out-of-sample performance of portfolios with a large number of stocks. Performance is measured in terms of portfolio volatility, Sharpe ratio, certainty-equivalent return, and turnover, with the benchmarks being the $1/N$ portfolio and three types of minimum-variance portfolios based on historical data: the first based on the sample covariance matrix, the second with short sale constraints, and the third with shrinkage applied to the covariance matrix.

We find that using the *option-implied volatilities*, even after correcting for the volatility-risk premium, does *not* improve portfolio performance significantly. The benefits from using *option-implied correlations* are even smaller. The reason for the small improvement in performance is that the estimates of implied volatilities and implied correlations are highly variable and give poorly behaved and unstable covariance matrices that then lead to highly-variable weights that fail to outperform the benchmarks.

We then adjust our estimates of expected returns on stocks using two sources of option-implied information. One, we use the *volatility risk premium* of each stock motivated by the empirical finding that stocks with high volatility risk premium tend to outperform those with low volatility risk premium. We start by setting expected returns across all stocks to be equal to one, while volatilities and correlations are estimated from historical data. Then, we rank all stocks according to their volatility risk premium and increase the estimate of expected returns by a factor for those stocks that are in the top decile of volatility risk premium and decrease it by the same factor for those in the bottom decile. Our empirical evidence shows that the portfolios where expected returns have been scaled using the volatility risk premium outperform the traditional portfolios in terms of Sharpe ratio and certainty-equivalent return, but with an increase in turnover. Two, we use the *model-free option-implied skewness*, to scale expected returns in the same manner as for the volatility risk premium. We find that portfolios based on implied skewness outperform even more strongly the traditional portfolios in terms of Sharpe ratio and certainty-equivalent return, but this increase is accompanied by an increase in turnover and portfolio volatility. We show that one way of reducing the impact on turnover is to use the volatility risk premium and implied skewness to scale stock-return volatilities rather than expected returns.

Based on our empirical analysis, we conclude that prices of stock options contain considerable information that can be used to improve the out-of-sample performance of mean-variance portfolios. In this paper, we have explored only very simple ways of incorporating information implied by option prices into static portfolios; more sophisticated ways of incorporating this information should lead to even larger gains in out-of-sample performance, with the challenge being how to do this while keeping turnover at a reasonable level.

A Computing (Co)variances from Intraday Data

Consistent with the literature on estimating moments from intraday data (see, for example, Brown (1990), Zhou (1996), and Corsi, Zumbach, Müller, and Dacorogna (2001)), we assume that instead of the true price, $X_i(t)$, we observe $Y_i(t)$ that is contaminated with noise; that is, $Y_i(t) = X_i(t) + \epsilon_i(t)$, where the noise process ϵ_i is assumed to be i.i.d and independent also of X_i. A common estimator for the integrated (co)variance of the efficient price process $\langle X_i, X_j \rangle$ is given by the *Realized (Co)Variance* (RV/RC):

$$\begin{aligned} \widehat{\langle Y_i, Y_j \rangle}^{(\Delta)} &= \sum_{k=1}^{n} r_i(k\Delta) \cdot r_j(k\Delta) \\ &= \sum_{k=1}^{n} \Big(Y_i\big(k\Delta\big) - Y_i\big((k-1)\cdot\Delta\big)\Big) \cdot \Big(Y_j\big(k\Delta\big) - Y_j\big((k-1)\cdot\Delta\big)\Big), \end{aligned} \qquad \text{(A1)}$$

where n is defined as $n = T/\Delta$ and $r_i(t)$ denotes the observed stock return for a time interval of length Δ; that is, $r_i(t) = Y_i(t) - Y_i(t - \Delta)$.

In the absence of microstructure noise, this estimator is consistent as the sampling frequency n increases (Jacod, 1994; Jacod and Protter, 1998). However, it is inconsistent under real market conditions where there is noise and asynchronous trading (Barndorff-Nielsen and Shephard, 2002; Zhang, Mykland, and Aït-Sahalia, 2005).[30] To mitigate this problem, we use the "second-best" estimator from Zhang, Mykland, and Aït-Sahalia (originally derived to estimate the realized variances) and apply it to realized (co)variances. The idea underlying this estimator is to compute the realized (co)variance estimator in (A1) at a low frequency in order to mitigate the problems induced by microstructure noise and non-synchronicity. When we sample at lower frequencies, we discard some observations and to overcome this problem Zhang, Mykland, and Aït-Sahalia suggest computing the realized (co)variance estimator in (A1) over different subsamples and then to average the estimators obtained for these subsamples. The "second-best" estimator is given by:

$$\widehat{\langle Y_i, Y_j \rangle}^{(avg,K)} = \frac{1}{K} \sum_{k=1}^{K} \widehat{\langle Y_i, Y_j \rangle}^{(\Delta,k)}. \qquad \text{(A2)}$$

We introduce one more averaging step to eliminate the chance of choosing the wrong sampling frequency by calculating the estimator (A2) over several frequencies and taking the mean. As our sample also includes less frequently traded stocks, especially early in the sample period, we choose relatively low sampling frequencies from 240 to 390 minutes (which corresponds to

[30] The non-synchronicity of the data induces an additional bias, known as the Epps effect (Epps (1979)), which drives covariances to zero as the sampling frequency increases.

the number of minutes in a typical trading day) with a step size of 10 minutes for the estimator (A2) to get the final realized (co)variance estimator:

$$\widehat{\langle Y_i, Y_j \rangle}^{(avg,\bar{K})} = \frac{1}{dim(\hat{K})} \sum_{s=1}^{dim(\bar{K})} \widehat{\langle Y_i, Y_j \rangle}^{(avg,\bar{K}(s))}. \tag{A3}$$

B Shrinkage and Regularization of Covariance Matrix

We consider a sample with a large number of stocks (100 and 561) and sample covariance matrices estimated from a limited history of daily stock returns are likely to be poorly behaved. To improve the sample covariance estimate for daily returns, we apply the optimal shrinkage methodology of Ledoit and Wolf (2004a,b):

$$\widehat{\Sigma}_{Shrunk} = (1-\phi)\widehat{\Sigma} + \phi S, \tag{B1}$$

where we shrink the sample estimate of the covariance matrix $\widehat{\Sigma}$ toward a diagonal matrix with the cross-sectional average variance on the diagonal, defined as the target S.[31] We minimize the Frobenius norm between the shrinkage estimator and the true covariance matrix in order to find the optimal shrinkage intensity parameter ϕ, using the time series of 750 points; technical details can be found in Ledoit and Wolf (2004a,b).

The asymptotic properties of intraday data are different from the asymptotic properties of the daily data, as shown by Zhang, Mykland, and Aït-Sahalia (2005) among others, and the intraday data do not satisfy the distributional assumptions of Ledoit and Wolf (2004a,b). Therefore, to improve the properties of the covariance matrix for intraday returns, we apply regularization of the inverse covariance proposed by Zumbach (2009). He uses the spectral decomposition of the covariance matrix estimator $\widehat{\Sigma}$, which is:

$$\widehat{\Sigma} = \sum_{n=1}^{N} \lambda_n V_n V_n', \tag{B2}$$

where $\{\lambda_1, ..., \lambda_N\}$ are eigenvalues and $\{V_1, ..., V_N\}$ are eigenvectors (pairwise orthogonal) for the set of N stocks. We order the eigenvalues by decreasing values, such that λ_1 is the largest eigenvalue. The inverse square root covariance can then be written as:

$$\widehat{\Sigma}^{-1/2} = \sum_{n=1}^{N} \frac{1}{\sqrt{\lambda_n}} V_n V_n', \tag{B3}$$

[31] We also used the cross-sectional average covariances matrix as the target, but the first target performs better out of sample.

where it is easy to see that for $\lambda_n \approx 0$ the singularity problem arises. To overcome this problem, we define

$$\widetilde{\Sigma}^{-1/2} = \sum_{n=1}^{k} \frac{1}{\sqrt{\lambda_n}} V_n V_n^{'} + \frac{1}{\sqrt{\lambda_{k+1}}} \sum_{n=k+1}^{N} V_n V_n^{'}, \tag{B4}$$

and use $\widetilde{\Sigma}$ as the estimator of the covariance matrix. This approach allows us to maintain the covariance structure by keeping all eigenvectors $\{V_1, ..., V_N\}$ and to eliminate the singularity problem by substituting all eigenvalues that are smaller than λ_{k+1} by $\lambda_{k+1} \neq 0$. We choose the parameter k so that the first k eigenvalues explain 75% of the overall variance.

C The Construction of the Risk-Neutral Implied Moments

The formulas in this appendix follow closely Bakshi, Kapadia, and Madan (2003) and are reproduced here only for completeness; for more details, please refer to the original paper. The approximation procedure for computing the integrals is our own.

Let $S(t)$ be the stock price at time t, $R(t,\tau)$ be the τ-period return (seen at time $t+\tau$) given by the log-price relative:

$$R(t,\tau) \equiv ln[S(t+\tau) - S(t)]. \tag{C1}$$

Let r be the interest rate, $C(t,\tau;K)$ and $P(t,\tau;K)$ the prices of call and put options written on the stock whose current price is $S(t)$, τ the time to maturity, and K the strike price.

Let $V(t,\tau) \equiv \mathcal{E}_t^*\{e^{-r\tau}R(t,\tau)^2\}$, $W(t,\tau) \equiv \mathcal{E}_t^*\{e^{-r\tau}R(t,\tau)^3\}$, and $X(t,\tau) \equiv \mathcal{E}_t^*\{e^{-r\tau}R(t,\tau)^4\}$ represent the fair value of the variance, cubic, and quartic contracts, respectively. Then, the price of the variance contract is given by

$$V(t,\tau) = \int_{S(t)}^{\infty} \frac{2\left(1-\log\left(\frac{K}{S(t)}\right)\right)}{K^2} \cdot C(t,\tau;K)dK + \int_{0}^{S(t)} \frac{2\left(1-\log\left(\frac{K}{S(t)}\right)\right)}{K^2} \cdot P(t,\tau;K)dK, \tag{C2}$$

the price of the cubic contract is

$$W(t,\tau) = \int_{S(t)}^{\infty} \frac{6\log\left(\frac{K}{S(t)}\right) - 3\left(\log\left(\frac{K}{S(t)}\right)\right)^2}{K^2} \cdot C(t,\tau;K)dK - \int_{0}^{S(t)} \frac{6\log\left(\frac{K}{S(t)}\right) + 3\left(\log\left(\frac{K}{S(t)}\right)\right)^2}{K^2} \cdot P(t,\tau;K)dK, \tag{C3}$$

and the price of the quartic contract is

$$X(t,\tau) = \int_{S(t)}^{\infty} \frac{12\left(ln[\frac{K}{S(t)}]\right)^2 - 4\left(ln[\frac{K}{S(t)}]\right)^3}{K^2} \cdot C(t,\tau;K)dK$$
$$+ \int_{0}^{S(t)} \frac{12\left(ln[\frac{S(t)}{K}]\right)^2 + 4\left(ln[\frac{S(t)}{K}]\right)^3}{K^2} \cdot P(t,\tau;K)dK. \quad \text{(C4)}$$

Define

$$\mu(t,\tau) = e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V(t,\tau) - \frac{e^{r\tau}}{6}W(t,\tau) - \frac{e^{r\tau}}{24}X(t,\tau).$$

Then, the τ-period model-free implied volatility (MFIV) can be calculated as

$$\text{MFIV}(t,\tau) = \left(V(t,\tau)\right)^{1/2}, \quad \text{(C5)}$$

and the τ-period model-free implied skewness (MFIS) as

$$\text{MFIS}(t,\tau) = \frac{e^{r\tau}W(t,\tau) - 3\mu(t,\tau)e^{r\tau}V(t,\tau) + 2(\mu(t,\tau))^3}{(e^{r\tau}V(t,\tau) - (\mu(t,\tau))^2)^{\frac{3}{2}}}. \quad \text{(C6)}$$

To calculate the integrals in (C2), (C3) and (C4) precisely, we need a continuum of option prices. We discretize the respective integrals and approximate them using the available options. As mentioned earlier, we normally have 13 out-of-the-money call and put implied volatilities for each maturity. Using cubic splines, we interpolate them inside the available moneyness range, and extrapolate using the last known (boundary for each side) value to fill in a total of 1001 grid points in the moneyness range from 1/3 to 3.[32] Then we calculate the option prices from the interpolated volatilities using the known interest rate for a given maturity, and use these prices to compute the model-free implied volatility and model-free implied skewness as in (C5) and (C6), respectively.

[32]The reason for choosing such a wide grid is that our simulation studies have shown that with a narrower grid we may not be estimating the skew and kurtosis of the risk-neutral distribution well enough. Decreasing the number of points in the grid also leads to a deterioration in accuracy.

Table 1: Benchmark portfolios that do not use option-implied information

In this table, we evaluate the performance of various benchmark portfolios that are based on historical returns and do *not* rely on prices of options. The $1/N$ portfolio is the equally-weighted strategy where one invests an equal amount of wealth across all N available stocks each period. The "Sample cov" portfolio is based on the sample covariance matrix; "Constrained" is the portfolio based on the sample covariance matrix but with short sales constrained; and, "Shrinkage" is the portfolio where shrinkage has been applied to the sample covariance matrix using the Ledoit and Wolf (2004a,b) methodology. We report two p-values in parenthesis, the first with respect to the $1/N$ strategy, and the second with respect to the "Sample cov" strategy, with the null hypothesis being that the strategy being evaluated is worse than the benchmark (so a small p-value suggests *rejecting* the null hypothesis that the policy being evaluated is worse than the benchmark).

	100 stocks				561 stocks			
Strategy	$\widehat{\sigma}$	sr	ce	trn	$\widehat{\sigma}$	sr	ce	trn
Panel A: Daily rebalancing								
1/N	0.1609	0.9286	0.1365	0.0129	0.1745	0.7489	0.1155	0.0144
Sample cov	0.1217	0.8089	0.0911	0.0849	0.1347	0.4737	0.0547	0.5360
	(0.00)	(0.67)	(0.87)		(0.00)	(0.78)	(0.86)	
	(0.50)	(0.50)	(0.50)		(0.50)	(0.50)	(0.50)	
Constrained	0.1242	0.7557	0.0862	0.0262	0.1240	0.9701	0.1126	0.0376
	(0.00)	(0.83)	(0.96)		(0.00)	(0.15)	(0.53)	
	(0.93)	(0.63)	(0.60)		(0.00)	(0.03)	(0.05)	
Shrinkage	0.1205	0.8514	0.0953	0.0745	0.1180	0.6142	0.0655	0.3046
	(0.00)	(0.61)	(0.86)		(0.00)	(0.66)	(0.84)	
	(0.00)	(0.05)	(0.09)		(0.00)	(0.06)	(0.20)	
Panel B: Monthly rebalancing								
1/N	0.1531	0.9402	0.1322	0.0595	0.1708	0.7339	0.1107	0.0673
Sample cov	0.1221	0.7903	0.0891	0.1762	0.1296	0.4768	0.0534	0.7785
	(0.00)	(0.74)	(0.91)		(0.00)	(0.82)	(0.91)	
	(0.50)	(0.50)	(0.50)		(0.50)	(0.50)	(0.50)	
Constrained	0.1207	0.7632	0.0848	0.0545	0.1180	1.0135	0.1126	0.0690
	(0.00)	(0.84)	(0.97)		(0.00)	(0.07)	(0.49)	
	(0.37)	(0.58)	(0.62)		(0.01)	(0.00)	(0.01)	
Shrinkage	0.1211	0.8280	0.0930	0.1602	0.1196	0.5842	0.0627	0.4979
	(0.00)	(0.69)	(0.89)		(0.00)	(0.73)	(0.90)	
	(0.13)	(0.03)	(0.05)		(0.00)	(0.09)	(0.17)	

Table 2: Prediction of variances and correlations

In Panel A of this table, we report the results of variance prediction regressions $RV = \alpha + \beta \widehat{RV}$, where we regress the 30-days realized variance (RV) on the variance predictors ($\widehat{RV}$). In Panel B, we show the results of the correlation prediction regressions $corr = \alpha + \beta \widehat{corr}$, where we regress the 30-days realized correlation on the correlation predictors ($\widehat{corr}$). We use 750 days of data to calculate the historical predictors. For covariances based on daily data, shrinkage is applied using the Ledoit and Wolf (2004a,b) methodology; for covariances based on intraday data, regularization is applied using the methodology in Zumbach (2009). In both cases, we infer the correlations from the covariances to which shrinkage/regularization has been applied. In parenthesis we report the p-values for the two-sided null hypotheses: $\alpha = 0$ and $\beta = 1$.

	100 stocks			561 stocks		
Predictor	α	β	R^2	α	β	R^2
Panel A: Prediction of variances						
Historical daily	0.0369	0.6795	0.2031	0.0520	0.6515	0.1966
	(0.03)	(0.05)		(0.10)	(0.09)	
Historical intraday	0.0616	0.5075	0.2308	0.0717	0.5667	0.2299
	(0.95)	(0.01)		(0.94)	(0.06)	
Implied variance	0.0214	0.7460	0.3334	0.0164	0.7793	0.3312
	(0.98)	(0.19)		(0.97)	(0.21)	
Implied variance (HVRP corrected)	0.0372	0.8189	0.3222	0.0435	0.8735	0.3155
	(0.97)	(0.31)		(0.96)	(0.31)	
Panel B: Prediction of correlations						
Historical daily	0.2007	0.0821	0.0238	0.2548	-0.1761	0.0397
	(0.84)	(0.02)		(0.80)	(0.07)	
Shrinkage daily	0.1987	0.0988	0.0245	0.2501	-0.0814	0.0328
	(0.84)	(0.03)		(0.80)	(0.04)	
Historical intraday	0.1703	0.2312	0.0680	0.1776	0.1880	0.0626
	(0.87)	(0.00)		(0.86)	(0.00)	
Shrinkage intraday	0.1899	1.4419	0.0419	0.2014	1.2203	0.0479
	(0.85)	(0.38)		(0.84)	(0.37)	
Implied correlation	0.1130	0.3528	0.0826	0.1116	0.3503	0.0932
	(0.91)	(0.00)		(0.91)	(0.01)	

Table 3: Portfolios using implied volatilities, historical correlations, and equal means

In this table, we evaluate the performance of various portfolios that use the model-free implied volatility calculated from option prices, but with correlations estimated from historical data, and with expected returns set equal across assets. The "Sample cov" portfolio is based on the sample covariance matrix but where historical volatility is replaced by the option-implied volatility; "Constrained" is the portfolio based on the same covariance matrix as above, but where short sales are constrained; and, "Shrinkage" is the portfolio based on the same covariance matrix as above, but with shrinkage applied to the covariance matrix using the Ledoit and Wolf (2004a,b) methodology. We report two p-values in parenthesis, the first with respect to the $1/N$ strategy, and the second with respect to the corresponding minimum-variance benchmark strategy in Table 1, with the null hypothesis being that the strategy being evaluated is worse than the benchmark (so a small p-value suggests *rejecting* the null hypothesis that the policy being evaluated is worse than the benchmark).

	100 stocks				561 stocks			
Strategy	$\widehat{\sigma}$	sr	ce	trn	$\widehat{\sigma}$	sr	ce	trn
				Panel A: Daily rebalancing				
1/N	0.1609	0.9286	0.1365	0.0129	0.1745	0.7489	0.1155	0.0144
Sample cov	0.1212	0.3724	0.0378	0.5801	0.1307	0.4792	0.0541	1.5504
	(0.00)	(0.98)	(0.99)		(0.00)	(0.81)	(0.89)	
	(0.39)	(0.99)	(0.99)		(0.19)	(0.49)	(0.50)	
Constrained	0.1210	0.6339	0.0694	0.2312	0.1240	0.6458	0.0724	0.2743
	(0.00)	(0.94)	(0.99)		(0.00)	(0.65)	(0.85)	
	(0.02)	(0.81)	(0.83)		(0.46)	(0.92)	(0.92)	
Shrinkage	0.1188	0.4174	0.0425	0.5154	0.1156	0.3953	0.0390	1.1032
	(0.00)	(0.98)	(0.99)		(0.00)	(0.90)	(0.96)	
	(0.14)	(0.99)	(1.00)		(0.17)	(0.84)	(0.85)	
				Panel B: Monthly rebalancing				
1/N	0.1531	0.9402	0.1322	0.0595	0.1708	0.7339	0.1107	0.0673
Sample cov	0.1139	0.5566	0.0569	0.6168	0.1251	0.5051	0.0555	1.6359
	(0.00)	(0.96)	(0.99)		(0.00)	(0.85)	(0.94)	
	(0.02)	(0.94)	(0.96)		(0.23)	(0.46)	(0.49)	
Constrained	0.1143	0.6745	0.0705	0.2402	0.1159	0.7628	0.0817	0.2832
	(0.00)	(0.96)	(1.00)		(0.00)	(0.41)	(0.84)	
	(0.01)	(0.80)	(0.87)		(0.36)	(0.92)	(0.93)	
Shrinkage	0.1132	0.5930	0.0607	0.5508	0.1161	0.4546	0.0461	1.1763
	(0.00)	(0.94)	(0.99)		(0.00)	(0.92)	(0.98)	
	(0.02)	(0.96)	(0.97)		(0.24)	(0.79)	(0.80)	

Table 4: Portfolios using risk-premium-corrected implied volatilities, historical correlations, and equal means

In this table, we evaluate the performance of various portfolios that use the risk-premium-corrected model-free implied volatility calculated from option prices. Correlations are estimated from historical data, and expected returns are set equal across assets. The "Sample cov" portfolio is based on the sample covariance matrix but where historical volatility is replaced by the risk-premium-corrected option-implied volatility; "Constrained" is the portfolio based on the same covariance matrix as above, but where short sales are constrained; and, "Shrinkage" is the portfolio based on the same covariance matrix as above, but with shrinkage applied to the covariance matrix using the Ledoit and Wolf (2004a,b) methodology. We report two p-values in parenthesis, the first with respect to the $1/N$ strategy, and the second with respect to the corresponding minimum-variance benchmark strategy in Table 1, with the null hypothesis being that the strategy being evaluated is worse than the benchmark (so a small p-value suggests *rejecting* the null hypothesis that the policy being evaluated is worse than the benchmark).

	100 stocks				561 stocks			
Strategy	$\widehat{\sigma}$	sr	ce	trn	$\widehat{\sigma}$	sr	ce	trn
				Panel A: Daily rebalancing				
1/N	0.1609	0.9286	0.1365	0.0129	0.1745	0.7489	0.1155	0.0144
Sample cov	0.1215	0.7735	0.0866	0.4782	0.1246	0.6837	0.0775	1.1788
	(0.00)	(0.72)	(0.90)		(0.00)	(0.58)	(0.78)	
	(0.46)	(0.58)	(0.58)		(0.03)	(0.23)	(0.27)	
Constrained	0.1226	0.7990	0.0904	0.2139	0.1180	0.6903	0.0745	0.2433
	(0.00)	(0.73)	(0.93)		(0.00)	(0.58)	(0.82)	
	(0.17)	(0.39)	(0.42)		(0.16)	(0.86)	(0.89)	
Shrinkage	0.1185	0.7787	0.0853	0.4183	0.1107	0.6047	0.0608	0.8444
	(0.00)	(0.72)	(0.91)		(0.00)	(0.72)	(0.91)	
	(0.12)	(0.67)	(0.69)		(0.00)	(0.53)	(0.58)	
				Panel B: Monthly rebalancing				
1/N	0.1531	0.9402	0.1322	0.0595	0.1708	0.7339	0.1107	0.0673
Sample cov	0.1136	0.9282	0.0990	0.5133	0.1191	0.7368	0.0807	1.2480
	(0.00)	(0.53)	(0.85)		(0.00)	(0.50)	(0.79)	
	(0.04)	(0.20)	(0.31)		(0.07)	(0.11)	(0.16)	
Constrained	0.1155	0.8509	0.0916	0.2231	0.1089	0.8863	0.0907	0.2496
	(0.00)	(0.69)	(0.94)		(0.00)	(0.20)	(0.73)	
	(0.10)	(0.26)	(0.34)		(0.12)	(0.74)	(0.83)	
Shrinkage	0.1119	0.9104	0.0956	0.4529	0.1126	0.6549	0.0674	0.9062
	(0.00)	(0.56)	(0.89)		(0.00)	(0.67)	(0.92)	
	(0.01)	(0.27)	(0.44)		(0.09)	(0.31)	(0.40)	

Table 5: Portfolios using historical volatilities, implied correlations, and equal means

In this table, we evaluate the performance of various portfolios that use option-implied correlations, as computed in Buss and Vilkov (2008). Volatilities are estimated from historical data, and expected returns are set equal across assets. The "Sample cov" portfolio is based on the sample covariance matrix but where option-implied correlations are used; "Constrained" is the portfolio based on the same covariance matrix as above, but where short sales are constrained; and, "Shrinkage" is the portfolio based on the same covariance matrix as above, but with shrinkage applied to the covariance matrix using the Ledoit and Wolf (2004a,b) methodology. We report two p-values in parenthesis, the first with respect to the $1/N$ strategy, and the second with respect to the corresponding minimum-variance benchmark strategy in Table 1, with the null hypothesis being that the strategy being evaluated is worse than the benchmark (so a small p-value suggests *rejecting* the null hypothesis that the policy being evaluated is worse than the benchmark).

	100 stocks				561 stocks			
Strategy	$\widehat{\sigma}$	sr	ce	trn	$\widehat{\sigma}$	sr	ce	trn
				Panel A: Daily rebalancing				
1/N	0.1609	0.9286	0.1365	0.0129	0.1745	0.7489	0.1155	0.0144
Sample cov	0.1462	0.2161	0.0209	0.4535	0.1302	0.3780	0.0407	0.4957
	(0.00)	(0.99)	(0.99)		(0.00)	(0.86)	(0.91)	
	(1.00)	(0.99)	(0.98)		(0.07)	(0.61)	(0.63)	
Constrained	0.1311	0.9001	0.1094	0.2109	0.1525	0.8086	0.1117	0.1044
	(0.00)	(0.55)	(0.83)		(0.00)	(0.28)	(0.59)	
	(1.00)	(0.15)	(0.10)		(1.00)	(0.79)	(0.52)	
Shrinkage	0.1357	0.3509	0.0384	0.2286	0.1344	0.2743	0.0278	0.3354
	(0.00)	(0.98)	(0.99)		(0.00)	(0.92)	(0.95)	
	(1.00)	(0.99)	(0.97)		(1.00)	(0.88)	(0.85)	
				Panel B: Monthly rebalancing				
1/N	0.1531	0.9402	0.1322	0.0595	0.1708	0.7339	0.1107	0.0673
Sample cov	0.1455	0.1660	0.0135	0.5098	0.1448	0.2666	0.0280	0.5825
	(0.20)	(0.99)	(1.00)		(0.02)	(0.90)	(0.93)	
	(1.00)	(1.00)	(1.00)		(0.95)	(0.80)	(0.78)	
Constrained	0.1231	0.8009	0.0910	0.2284	0.1450	0.8488	0.1125	0.1445
	(0.00)	(0.81)	(0.96)		(0.00)	(0.07)	(0.46)	
	(0.71)	(0.41)	(0.36)		(1.00)	(0.81)	(0.50)	
Shrinkage	0.1428	0.2545	0.0261	0.2820	0.1504	0.1441	0.0102	0.4188
	(0.12)	(1.00)	(1.00)		(0.06)	(0.95)	(0.96)	
	(1.00)	(1.00)	(1.00)		(1.00)	(0.97)	(0.95)	

Table 6: Portfolio using historical volatilities, historical correlations, and expected returns scaled by volatility risk premium

In this table, we evaluate the performance of portfolios computed with expected returns that have been scaled by the volatility risk premium using the following procedure. We first set expected returns for all assets equal to 1. We then sort all stocks by the characteristic "volatility risk premium" into deciles, and then change the expected returns of the top decile to $(1 + \delta)$ and of the bottom decile to $(1 - \delta)$, with $\delta_1 = 0.50$ and $\delta_2 = 0.70$. The "Sample cov" portfolio is based on the sample covariance matrix and the scaled expected returns; "Constrained" is based on the same expected returns and covariance matrix but where short sales are constrained; and, "Shrinkage" is based on the same expected returns but with shrinkage applied to the sample covariance matrix. We report two p-values in parenthesis, the first with respect to the $1/N$ strategy, and the second with respect to the corresponding minimum-variance benchmark strategy in Table 1, with the null hypothesis being that the strategy being evaluated is worse than the benchmark.

Panel A: Daily rebalancing

	100 stocks				561 stocks			
Strategy	$\widehat{\sigma}$	sr	ce	trn	$\widehat{\sigma}$	sr	ce	trn
1/N	0.1609	0.9286	0.1365	0.0129	0.1745	0.7489	0.1155	0.0144
Sample cov: δ_1	0.1504	1.1289	0.1585	0.2648	0.1883	0.1908	0.0182	1.5386
	(0.00)	(0.27)	(0.34)		(1.00)	(0.93)	(0.92)	
	(1.00)	(0.06)	(0.01)		(1.00)	(0.86)	(0.79)	
Sample cov: δ_2	0.1706	1.1448	0.1807	0.3411	0.2214	0.1314	0.0047	1.9668
	(0.99)	(0.28)	(0.24)		(1.00)	(0.94)	(0.91)	
	(1.00)	(0.10)	(0.01)		(1.00)	(0.87)	(0.80)	
Constrained: δ_1	0.1392	1.0921	0.1423	0.0407	0.1297	0.9469	0.1144	0.0400
	(0.00)	(0.23)	(0.43)		(0.00)	(0.15)	(0.52)	
	(1.00)	(0.03)	(0.01)		(1.00)	(0.56)	(0.47)	
Constrained: δ_2	0.1393	1.1004	0.1436	0.0420	0.1270	0.9759	0.1159	0.0517
	(0.00)	(0.22)	(0.42)		(0.00)	(0.15)	(0.50)	
	(1.00)	(0.03)	(0.01)		(0.95)	(0.49)	(0.44)	
Shrinkage: δ_1	0.1432	1.1696	0.1573	0.2272	0.1506	0.3117	0.0356	0.8830
	(0.00)	(0.23)	(0.34)		(0.00)	(0.88)	(0.90)	
	(1.00)	(0.06)	(0.01)		(1.00)	(0.91)	(0.82)	
Shrinkage: δ_2	0.1606	1.1889	0.1780	0.2924	0.1722	0.2287	0.0246	1.1268
	(0.47)	(0.23)	(0.23)		(0.34)	(0.91)	(0.91)	
	(1.00)	(0.09)	(0.01)		(1.00)	(0.93)	(0.84)	

Panel B: Monthly rebalancing

	100 stocks				561 stocks			
Strategy	$\widehat{\sigma}$	sr	ce	trn	$\widehat{\sigma}$	sr	ce	trn
1/N	0.1531	0.9402	0.1322	0.0595	0.1708	0.7339	0.1107	0.0673
Sample cov: δ_1	0.1504	0.9612	0.1333	0.4012	0.1945	0.2015	0.0203	1.9049
	(0.40)	(0.47)	(0.48)		(0.93)	(0.95)	(0.93)	
	(1.00)	(0.18)	(0.04)		(1.00)	(0.91)	(0.80)	
Sample cov: δ_2	0.1714	0.9522	0.1485	0.5136	0.2320	0.1484	0.0076	2.4532
	(0.95)	(0.49)	(0.37)		(1.00)	(0.96)	(0.93)	
	(1.00)	(0.24)	(0.05)		(1.00)	(0.91)	(0.81)	
Constrained: δ_1	0.1375	1.0237	0.1314	0.0695	0.1368	0.9047	0.1144	0.0754
	(0.03)	(0.36)	(0.51)		(0.00)	(0.16)	(0.45)	
	(1.00)	(0.08)	(0.02)		(1.00)	(0.80)	(0.46)	
Constrained: δ_2	0.1372	1.0283	0.1318	0.0706	0.1336	0.9374	0.1163	0.0816
	(0.03)	(0.37)	(0.52)		(0.00)	(0.15)	(0.43)	
	(1.00)	(0.08)	(0.02)		(1.00)	(0.72)	(0.42)	
Shrinkage: δ_1	0.1433	1.0042	0.1336	0.3515	0.1634	0.3026	0.0362	1.1481
	(0.16)	(0.42)	(0.49)		(0.30)	(0.93)	(0.93)	
	(1.00)	(0.17)	(0.04)		(1.00)	(0.93)	(0.81)	
Shrinkage: δ_2	0.1614	0.9971	0.1479	0.4472	0.1897	0.2303	0.0259	1.4623
	(0.79)	(0.44)	(0.37)		(0.87)	(0.95)	(0.93)	
	(1.00)	(0.23)	(0.04)		(1.00)	(0.94)	(0.82)	

Table 7: Portfolios using historical volatilities scaled by volatility risk premium, historical correlations, and equal means

In this table, we evaluate the performance of portfolios computed with historical volatilities that have been scaled by the volatility risk premium using the following procedure. We sort all stocks by the characteristic "volatility risk premium" into deciles, and then change the volatility of the top decile to $(1-\delta)$ and of the bottom decile to $(1+\delta)$, with $\delta_1 = 0.50$ and $\delta_2 = 0.70$. Correlations are estimated from historical data and expected returns for all assets are set equal to 1. The "Sample cov" portfolio is based on the adjusted sample covariance matrix with volatilities scaled as described above; "Constrained" is based on the same adjusted covariance matrix but where short sales are constrained; and, "Shrinkage" is based on the same adjusted covariance matrix but with shrinkage applied to the sample covariance matrix. We report two p-values in parenthesis, the first with respect to the $1/N$ strategy, and the second with respect to the corresponding minimum-variance benchmark strategy in Table 1, with the null hypothesis being that the strategy being evaluated is worse than the benchmark.

	100 stocks				561 stocks			
Strategy	$\widehat{\sigma}$	sr	ce	trn	$\widehat{\sigma}$	sr	ce	trn
Panel A: Daily rebalancing								
1/N	0.1609	0.9286	0.1365	0.0129	0.1745	0.7489	0.1155	0.0144
Sample cov: δ_1	0.1400	1.2109	0.1598	0.1213	0.1343	0.7288	0.0889	0.4248
	(0.00)	(0.13)	(0.26)		(0.00)	(0.54)	(0.76)	
	(1.00)	(0.03)	(0.01)		(0.44)	(0.18)	(0.18)	
Sample cov: δ_2	0.1442	1.1922	0.1616	0.0876	0.1354	0.7624	0.0941	0.2492
	(0.00)	(0.13)	(0.24)		(0.00)	(0.47)	(0.74)	
	(1.00)	(0.06)	(0.02)		(0.57)	(0.18)	(0.17)	
Constrained: δ_1	0.1384	0.9465	0.1214	0.0470	0.1480	0.9401	0.1282	0.0358
	(0.00)	(0.45)	(0.72)		(0.00)	(0.03)	(0.25)	
	(1.00)	(0.09)	(0.04)		(1.00)	(0.55)	(0.29)	
Constrained: δ_2	0.1398	1.0049	0.1307	0.0326	0.1492	0.8680	0.1184	0.0343
	(0.00)	(0.32)	(0.58)		(0.00)	(0.08)	(0.43)	
	(1.00)	(0.05)	(0.01)		(1.00)	(0.70)	(0.41)	
Shrinkage: δ_1	0.1378	1.2269	0.1596	0.1092	0.1299	0.7648	0.0909	0.2750
	(0.00)	(0.11)	(0.26)		(0.00)	(0.47)	(0.76)	
	(1.00)	(0.03)	(0.01)		(1.00)	(0.27)	(0.21)	
Shrinkage: δ_2	0.1431	1.2049	0.1622	0.0808	0.1344	0.7797	0.0958	0.1673
	(0.00)	(0.11)	(0.23)		(0.00)	(0.43)	(0.74)	
	(1.00)	(0.06)	(0.02)		(1.00)	(0.28)	(0.20)	
Panel B: Monthly rebalancing								
1/N	0.1531	0.9402	0.1322	0.0595	0.1708	0.7339	0.1107	0.0673
Sample cov: δ_1	0.1381	1.0961	0.1419	0.1841	0.1371	0.7447	0.0928	0.5157
	(0.04)	(0.28)	(0.39)		(0.00)	(0.48)	(0.71)	
	(0.99)	(0.07)	(0.02)		(0.82)	(0.09)	(0.07)	
Sample cov: δ_2	0.1416	1.0993	0.1457	0.1368	0.1370	0.7903	0.0990	0.3068
	(0.08)	(0.26)	(0.35)		(0.00)	(0.38)	(0.65)	
	(1.00)	(0.08)	(0.03)		(0.83)	(0.07)	(0.06)	
Constrained: δ_1	0.1323	0.9444	0.1162	0.0809	0.1469	0.8935	0.1204	0.0819
	(0.00)	(0.48)	(0.77)		(0.00)	(0.01)	(0.22)	
	(1.00)	(0.12)	(0.04)		(1.00)	(0.75)	(0.37)	
Constrained: δ_2	0.1340	1.0177	0.1274	0.0677	0.1492	0.8708	0.1187	0.0813
	(0.00)	(0.31)	(0.59)		(0.00)	(0.01)	(0.24)	
	(1.00)	(0.07)	(0.02)		(1.00)	(0.79)	(0.40)	
Shrinkage: δ_1	0.1359	1.1119	0.1419	0.1699	0.1371	0.7507	0.0936	0.3585
	(0.02)	(0.25)	(0.39)		(0.00)	(0.46)	(0.71)	
	(0.99)	(0.08)	(0.03)		(1.00)	(0.16)	(0.09)	
Shrinkage: δ_2	0.1406	1.1088	0.1461	0.1295	0.1393	0.7803	0.0990	0.2245
	(0.07)	(0.24)	(0.33)		(0.00)	(0.41)	(0.66)	
	(1.00)	(0.11)	(0.04)		(1.00)	(0.14)	(0.07)	

Table 8: Portfolios using historical volatilities, historical correlations, and expected returns scaled by implied skewness

In this table, we evaluate the performance of various portfolios computed with expected returns that have been scaled by option-implied skewness using the following procedure. We first set expected returns for all assets equal to 1. We then sort all the stocks by the characteristic "model free implied skewness" into deciles, and then change the expected returns of the top decile to $(1 + \delta)$ and of the bottom decile to $(1 - \delta)$, with $\delta_1 = 0.30$ and $\delta_2 = 0.50$. The "Sample cov" portfolio is based on the sample covariance matrix and the scaled expected returns; "Constrained" is based on the same expected returns and covariance matrix, but with short sales constrained; and, "Shrinkage" is based on the same expected returns but with shrinkage applied to the sample covariance matrix. We report two p-values in parenthesis, the first with respect to the $1/N$ strategy, and the second with respect to the corresponding minimum-variance benchmark strategy in Table 1, with the null hypothesis being that the strategy being evaluated is worse than the benchmark.

	100 stocks				561 stocks			
Strategy	$\widehat{\sigma}$	sr	ce	trn	$\widehat{\sigma}$	sr	ce	trn
				Panel A: Daily rebalancing				
1/N	0.1609	0.9286	0.1365	0.0129	0.1745	0.7489	0.1155	0.0144
Sample cov: δ_1	0.1416	1.4957	0.2018	0.8217	0.1765	1.3157	0.2167	3.5401
	(0.00)	(0.03)	(0.07)		(0.67)	(0.06)	(0.06)	
	(1.00)	(0.00)	(0.00)		(1.00)	(0.00)	(0.00)	
Sample cov: δ_2	0.1710	1.6974	0.2756	1.3684	0.2350	1.4775	0.3196	5.9327
	(0.99)	(0.01)	(0.00)		(1.00)	(0.03)	(0.01)	
	(1.00)	(0.00)	(0.00)		(1.00)	(0.00)	(0.00)	
Constrained: δ_1	0.1706	1.4907	0.2398	0.3609	0.1506	1.3195	0.1874	0.3373
	(1.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.02)	
	(1.00)	(0.00)	(0.00)		(1.00)	(0.05)	(0.01)	
Constrained: δ_2	0.1710	1.5013	0.2421	0.3567	0.1584	1.3557	0.2022	0.3158
	(1.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)	
	(1.00)	(0.00)	(0.00)		(1.00)	(0.03)	(0.00)	
Shrinkage: δ_1	0.1367	1.5020	0.1960	0.6957	0.1435	1.5803	0.2165	2.0959
	(0.00)	(0.02)	(0.08)		(0.00)	(0.01)	(0.03)	
	(1.00)	(0.00)	(0.00)		(1.00)	(0.00)	(0.00)	
Shrinkage: δ_2	0.1607	1.7152	0.2627	1.1549	0.1799	1.8332	0.3136	3.4775
	(0.47)	(0.01)	(0.00)		(0.88)	(0.00)	(0.00)	
	(1.00)	(0.00)	(0.00)		(1.00)	(0.00)	(0.00)	
				Panel B: Monthly rebalancing				
1/N	0.1531	0.9402	0.1322	0.0595	0.1708	0.7339	0.1107	0.0673
Sample cov: δ_1	0.1491	1.1166	0.1555	0.8796	0.1625	0.9437	0.1403	3.6817
	(0.33)	(0.24)	(0.26)		(0.26)	(0.23)	(0.26)	
	(1.00)	(0.00)	(0.00)		(1.00)	(0.00)	(0.00)	
Sample cov: δ_2	0.1823	1.1821	0.1993	1.4402	0.2158	1.0027	0.1934	6.1276
	(1.00)	(0.18)	(0.05)		(1.00)	(0.18)	(0.06)	
	(1.00)	(0.01)	(0.00)		(1.00)	(0.01)	(0.00)	
Constrained: δ_1	0.1657	1.0494	0.1602	0.3754	0.1656	0.8551	0.1278	0.3546
	(0.98)	(0.23)	(0.10)		(0.22)	(0.23)	(0.26)	
	(1.00)	(0.04)	(0.00)		(1.00)	(0.86)	(0.27)	
Constrained: δ_2	0.1657	1.0580	0.1617	0.3723	0.1817	0.8828	0.1440	0.3404
	(0.98)	(0.19)	(0.08)		(0.93)	(0.15)	(0.09)	
	(1.00)	(0.04)	(0.00)		(1.00)	(0.80)	(0.15)	
Shrinkage: δ_1	0.1431	1.1456	0.1538	0.7503	0.1421	1.1094	0.1477	2.2062
	(0.12)	(0.19)	(0.26)		(0.00)	(0.07)	(0.16)	
	(1.00)	(0.00)	(0.00)		(1.00)	(0.00)	(0.00)	
Shrinkage: δ_2	0.1702	1.2215	0.1937	1.2201	0.1764	1.2357	0.2026	3.6091
	(0.97)	(0.13)	(0.05)		(0.69)	(0.03)	(0.01)	
	(1.00)	(0.00)	(0.00)		(1.00)	(0.00)	(0.00)	

Table 9: Portfolios using historical volatilities scaled by implied skewness, historical correlations, and equal means

In this table, we evaluate the performance of portfolios computed with historical volatilities that have been scaled by the option-implied skewness using the following procedure. We sort all stocks by the characteristic "model free implied skewness" into deciles, and then change the volatility of the top decile to $(1-\delta)$ and of the bottom decile to $(1+\delta)$, with $\delta_1 = 0.30$ and $\delta_2 = 0.50$. Expected returns for all assets are set equal to 1. The "Sample cov" portfolio is based on the adjusted sample covariance matrix with volatilities scaled as described above; "Constrained" is based on the same adjusted covariance matrix but where short sales are constrained; and, "Shrinkage" is based on the same adjusted covariance matrix but with shrinkage applied to the sample covariance matrix. We report two p-values in parenthesis, the first with respect to the $1/N$ strategy, and the second with respect to the corresponding minimum-variance benchmark strategy in Table 1, with the null hypothesis being that the strategy being evaluated is worse than the benchmark.

	100 stocks				561 stocks			
Strategy	$\widehat{\sigma}$	sr	ce	trn	$\widehat{\sigma}$	sr	ce	trn
	Panel A: Daily rebalancing							
1/N	0.1609	0.9286	0.1365	0.0129	0.1745	0.7489	0.1155	0.0144
Sample cov: δ_1	0.1417	1.6164	0.2190	0.6574	0.1410	1.4079	0.1886	1.8965
	(0.00)	(0.00)	(0.01)		(0.00)	(0.01)	(0.04)	
	(1.00)	(0.00)	(0.00)		(0.99)	(0.00)	(0.00)	
Sample cov: δ_2	0.1663	1.7304	0.2740	0.7509	0.1543	1.4576	0.2130	1.5829
	(0.96)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)	
	(1.00)	(0.00)	(0.00)		(1.00)	(0.00)	(0.00)	
Constrained: δ_1	0.1439	1.1196	0.1507	0.2108	0.1651	0.8075	0.1197	0.0300
	(0.00)	(0.05)	(0.22)		(0.00)	(0.11)	(0.32)	
	(1.00)	(0.01)	(0.00)		(1.00)	(0.79)	(0.42)	
Constrained: δ_2	0.1517	1.2464	0.1776	0.2683	0.1653	0.7923	0.1173	0.0491
	(0.00)	(0.02)	(0.05)		(0.00)	(0.22)	(0.42)	
	(1.00)	(0.00)	(0.00)		(1.00)	(0.80)	(0.44)	
Shrinkage: δ_1	0.1394	1.6156	0.2155	0.5788	0.1319	1.5254	0.1926	1.2677
	(0.00)	(0.00)	(0.01)		(0.00)	(0.00)	(0.02)	
	(1.00)	(0.00)	(0.00)		(1.00)	(0.00)	(0.00)	
Shrinkage: δ_2	0.1647	1.7379	0.2727	0.6838	0.1527	1.5133	0.2194	1.1407
	(0.89)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)	
	(1.00)	(0.00)	(0.00)		(1.00)	(0.00)	(0.00)	
	Panel B: Monthly rebalancing							
1/N	0.1531	0.9402	0.1322	0.0595	0.1708	0.7339	0.1107	0.0673
Sample cov: δ_1	0.1454	1.1664	0.1592	0.6949	0.1448	1.0174	0.1370	1.9612
	(0.18)	(0.13)	(0.17)		(0.00)	(0.08)	(0.20)	
	(1.00)	(0.00)	(0.00)		(1.00)	(0.00)	(0.00)	
Sample cov: δ_2	0.1682	1.1906	0.1864	0.7791	0.1653	1.0349	0.1576	1.6204
	(0.97)	(0.07)	(0.02)		(0.24)	(0.04)	(0.04)	
	(1.00)	(0.01)	(0.00)		(1.00)	(0.00)	(0.00)	
Constrained: δ_1	0.1401	1.0323	0.1348	0.2381	0.1628	0.7662	0.1114	0.0806
	(0.00)	(0.15)	(0.42)		(0.00)	(0.12)	(0.43)	
	(1.00)	(0.03)	(0.00)		(1.00)	(0.91)	(0.51)	
Constrained: δ_2	0.1499	1.0601	0.1477	0.2919	0.1617	0.8249	0.1203	0.0982
	(0.24)	(0.13)	(0.16)		(0.00)	(0.01)	(0.09)	
	(1.00)	(0.03)	(0.00)		(1.00)	(0.85)	(0.39)	
Shrinkage: δ_1	0.1427	1.1856	0.1591	0.6159	0.1432	1.0670	0.1426	1.3281
	(0.08)	(0.09)	(0.15)		(0.00)	(0.04)	(0.13)	
	(1.00)	(0.00)	(0.00)		(1.00)	(0.00)	(0.00)	
Shrinkage: δ_2	0.1662	1.2104	0.1876	0.7124	0.1705	1.0499	0.1647	1.1787
	(0.96)	(0.04)	(0.01)		(0.49)	(0.02)	(0.02)	
	(1.00)	(0.01)	(0.00)		(1.00)	(0.01)	(0.00)	

Figure 1: Volatilities: Realized, historical, implied, and risk-premium-corrected implied

In this figure, we plot the historical volatility based on the past 750 days (dot-dashed blue line), model-free implied volatility (dashed red line), risk-premium-corrected model-free implied volatility (solid pink line), and the 30-day realized volatility (thick black line). The figure is based on the cross-sectional equally-weighted average volatilities across 561 stocks. The figure shows that risk-premium-correct implied volatility tracks realized volatility quite closely. The model-free implied volatility (without any risk-premium correction) tracks realized volatility, but there is a distinct gap between the two. And, the historical 750-day volatility does not track realized volatility closely. Note also that all these volatility series have different levels of variability: the implied and risk-premium-corrected implied volatilities are slightly more variable than the realized volatility, while historical volatility is the smoothest.

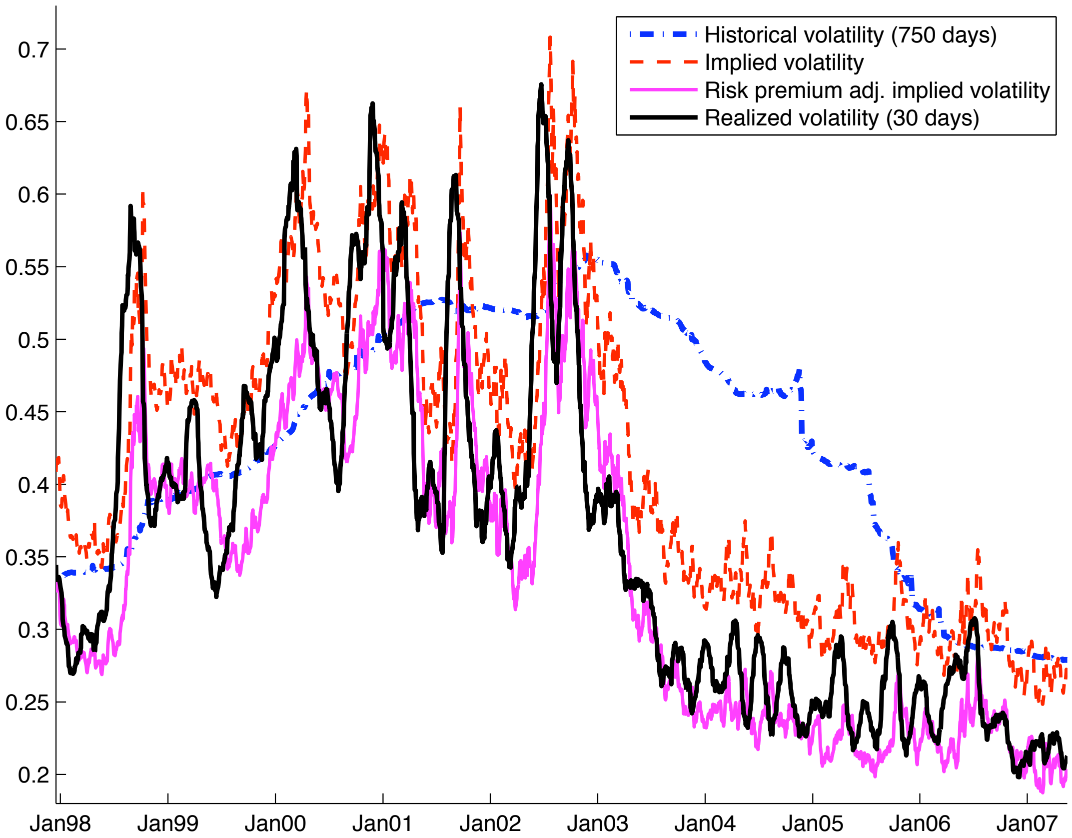
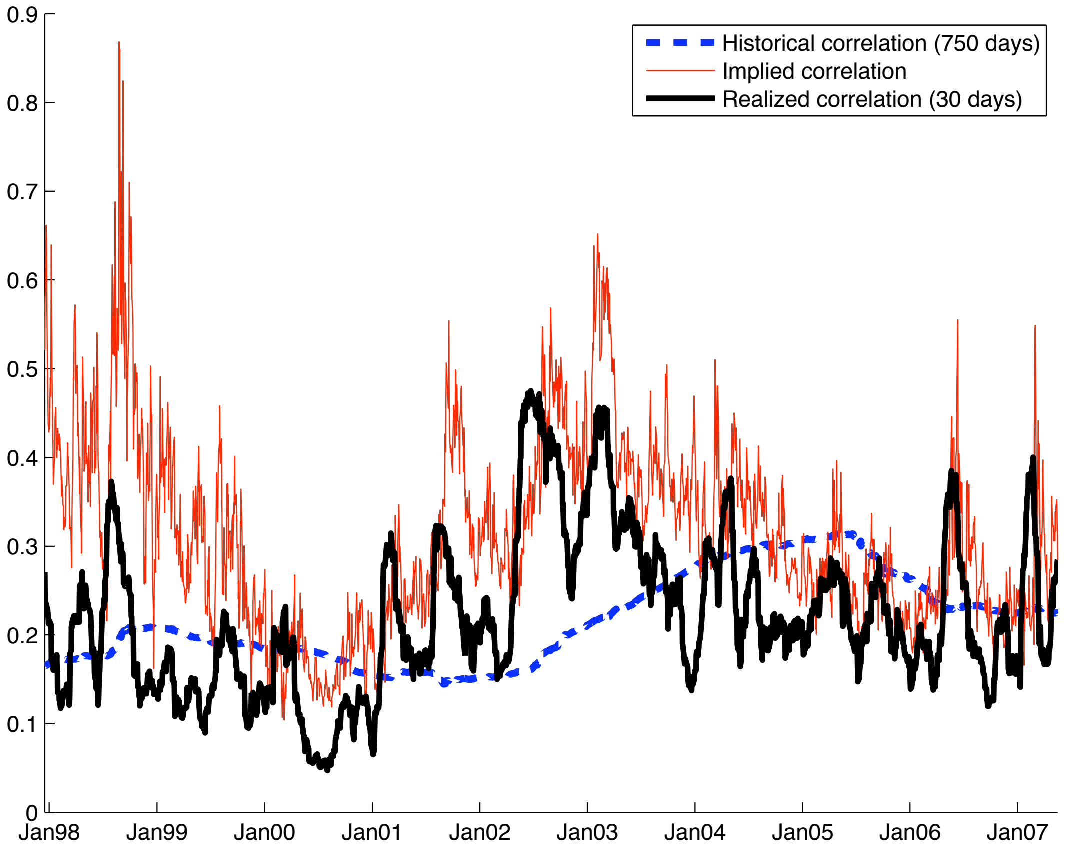


Figure 2: Correlations: Realized, historical, and implied

In this figure, we plot the historical correlation based on the past 750 days (dashed blue line), implied correlation (solid red line), and 30-day realized correlations (thick black line). The plot is based on the cross-sectional equally-weighted average of average correlations across 561 stocks. There are two observations about these series: first, implied correlation follows the level of realized correlation much more closely than historical correlation; two, implied correlation is much more volatile than realized correlation, while historical correlation is even smoother.



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